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X-RAY SPECTRA.

A Discussion on this subject was held on May 28th, 1920.

Sir W. H. BRAGG, F.R.S., President, who opened the discussion, said: The X-ray spectrometer, although but a few years old, is an instrument of very considerable precision. I propose to discuss briefly the nature of the measurements that can be made with it, the means of ensuring accuracy, certain results that have been obtained, and a few notable features of those results.

The outbreak of war practically put a stop to the work with the spectroscope, which had been commenced in England, and we have fallen behind other countries which have been able to push on with it. I think it is not generally known how far the subject has advanced abroad, and I hope that the brief discussion which has been arranged for this afternoon may serve to bring its fascinations before members who have not yet had the opportunity of examining it.

In the first place, let us recall that the spectrometer gives the angle at which X-rays of a given wave-length are reflected by a given set of planes belonging to a crystal. The quantities are connected by the relation $n\lambda=2d \sin \theta$, where λ is the wave-length, d the spacing of the planes, θ the reflecting angle, and n an integer representing the order of the spectrum. The spectrometer gives θ, and hence the ratio of λ to d can be calculated at once; but the spectrometer does not find λ or d separately; it gives angles, not lengths. It yields, however, so much information about the relative spacings of the different sets of planes in a crystal that we can, taking also into account the information given by a study of the crystal form, determine the architecture of the crystal, at any rate in the case of such simple crystals as rock-salt, diamond, calcite and so on.

When we know the structure, and if we assume values for the actual weights of the different atoms of which the crystal is composed, and if we also know the specific gravity of the crystal, we can find the spacing of any given set of planes. It is, by the way, convenient to express spacings in terms of the Angstrom unit (10^{-8}cm.). Into these determinations the

accuracy of the spectrometer does not enter at all. No more than a rough observation with the spectrometer is required to give the structure, and the remainder of the calculation is concerned entirely with quantities with which the spectrometer has nothing to do—viz., the specific gravity and the weights of the atoms in grams.

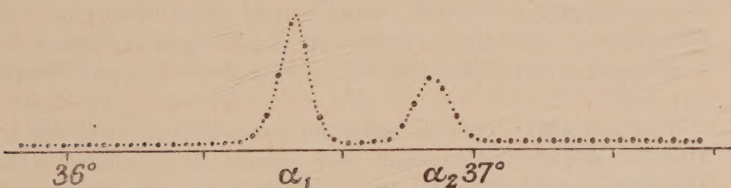
Let us remember that the weights of the atoms are found by measuring first the ratio of the charge of an electron to the mass of the ion in electrolysis, and secondly the charge on the electron itself. The best determination of the latter is probably that by Millikan, who used the oil-drop method—viz., $(4{\cdot}774 \pm 0{\cdot}005) \times 10^{-10}$ in electrostatic units. The derived value of the weight of the H atom is $(1{\cdot}662 \pm 0{\cdot}002) \times 10^{-24}$.

The accuracy with which the spectrometer angle can be measured is probably much greater than the accuracy with which the weight of the atom can be found. Fortunately, a knowledge of the angle alone is sufficient for most purposes. Every observer who uses the same kind of crystal starts on the same basis, for the spacing of a crystal depends on the properties of the atoms of which it is composed, and must be the same from whatever source the crystal comes.

Observers do not generally leave their results in terms of spectrometer angles, but assume a spacing for the crystal planes and then calculate wave-length or frequency. This is quite convenient for comparison so long as the same value of the spacing is always used. The crystals in most frequent use are rock salt and Iceland spar. It would be convenient if all workers assumed that the spacing of Na.Cl. (100) was 2·814 A.U. and of $CaCO_3$ (100) was 3·029 A.U. They are no doubt slightly in error—more so absolutely than relatively to each other; but, for most purposes, only relative values of wave-lengths are wanted, and it is more important that every one should have the same standard of reference than that it should be highly accurate.

The spectrometer may be fitted with an ionisation chamber or with a photographic plate. Each method has its advantages. The former gives the angle of reflection directly in terms of the angular graduations of the spectrometer, and involves no determinations of length. A fine pencil of X-rays falls on the crystal, which is revolved about an axis contained by the planes under investigation, until the reflection flashes out; observations of the intensity of the action in the ionisation chamber are made at short intervals, and the maximum

point is found. This can be done to a few seconds of arc. The figure shows observations of the lines $K\alpha_1$, $K\alpha_2$ in the rhodium spectrum of the fourth order, diamond being the crystal employed. The angles of reflection lie between 36° and 37°. Any zero of scale error is eliminated by making a corresponding set of observations on the other side. The ionisation chamber is not moved during either set; but, placed in its proper position, is left at rest with a wide open slit. The photographic method gives the positions of lines and absorption edges on a plate; and it is necessary to know the distance between the point of reflection in the crystal and the plate before the angle of reflection can be found. There is an uncertainty in the position of the point of reflection; it is, in fact, no point at all, because the rays are not reflected exactly at the surface of the crystal, but penetrate some



$K\alpha_1$ & $K\alpha_2$ of Rhodium in diamond : 4th order.

distance within. In order to get over this difficulty, Dershem ("Physical Review," June, 1918, p. 461) and Overn ("Physical Review," August, 1919, p. 137) have used extremely thin crystals. Siegbahn ("Phil. Mag.," November 19, p. 639) has used a fairly thin crystal of rock salt placed at right angles to the ray, so that the reflecting planes are in the body of the crystal and are of small area. Uhler and Cooksey ("Physical Review," December, 1917, p. 645) have observed the right and left images for two positions of the plate, first when the plate is a short distance, say 10 cm., from the crystal, and secondly when it has been withdrawn 70 cm. The amount of withdrawal must be exactly known, and can be conveniently regulated by a distance piece, which can afterwards be measured by the same apparatus that is used to measure the separation of the images on the plates. It is easy to calculate the angle of reflection from these data.

The photographic method has the advantage of leaving a permanent record, and it is cumulative, since it can be left in

operation for hours, during which no observations are required. An observable result is obtained much more quickly with the ionisation chamber than with the photographic plate. I am not aware of any careful comparison, but, from rough measurement of my own, the former is of the order of a thousand times the sensitivity of the latter. But the ionisation method requires a number of successive measurements over separate points in the spectrum in order to make a survey of the region in question, whereas one photograph is alone required. The former method requires greater care in avoiding errors due to fluctuation of the X-ray bulb and to imperfect insulation of the chamber or the electroscope; the latter requires more power. The ionisation method is far more reliable for the purpose of comparing the intensities of different lines, and is also, I would suppose, more accurate in determining their position, since the scale is so much more open. The scale of the figure given above is certainly not more open than the observations warrant, whereas in a photograph deflections ranging over one degree are compressed into a few millimetres at most.

The ionisation method has been used with fine results by Duane, Webster, Compton and others in America; but Dershem, Overn and Hull have used photography. The work of these observers is to be found for the most part in the "Physical Review." Photography has also been used by many other observers doing excellent work, as, for instance, by de Broglie, whose papers are to be found in the "Comptes Rendus"; by Wagner, by Debye; also by Siegbahn and his collaborators at the University of Lund, who contribute to the "Philosophical Magazine." The refined apparatus of the latter is described in the "Philosophical Magazine" of June and November, 1919.

An X-ray spectrum shows :—

1. Fine lines due to the material of the anti-cathode. It is usually intended that the anti-cathode shall be of pure material, impurities may show as faint lines imposed on the true spectrum. There may also be faint lines due to monochromatic rays scattered by the anti-cathode, but not originally produced there.

2. Absorption effects due to the substances traversed by the rays. These are of two kinds :—

(*a*) Any substance traversed by the rays on their way to the plate, such as absorbing screens intentionally placed there,

or gases, or the wall of the bulb, or the crystal, or even the substance of the anti-cathode itself will absorb the rays on the short side of certain critical wave-lengths more than the rays on the long side ; the critical wave-lengths are characteristic of the atoms passed through. Consequently, the plate shows a darkening (intenser action) on the *long* wave-length side ; the dividing line may be referred to as an absorption edge.

(*b*) Any substance in the photographic film itself or sufficiently near to it will, owing to this same absorption effect, cause more action on the film on the *short* side of a critical wave-length than on the *long*, because high absorption means excessive production of cathode rays and it is these that affect the plate.

There are no absorption lines, or bands properly so called ; unless the term can be applied to certain narrow fluctuations of intensity close to an absorption edge. These have been announced as observable in certain cases ; but the edge is usually so sharp as to deserve the name of a discontinuity.

The lines due to a given substance used as anti-cathode are, as is well known, divided into series, K, L and M, the first-named consisting of the shortest wave-lengths. These constitute the characteristic radiations discovered by Barkla. From certain indirect evidence the existence of a J series, shorter than the K, has been presumed ; but this is not confirmed by a direct examination (Duane and Shimizu, " Physical Review," November, 1919, p. 389).

The subjoined table is compiled from the results of a few of the workers in this subject, and is arranged to show generally the wave-lengths and intensities of the lines in the K and L series, so far as they have been observed, and incidentally the extent of the concordance between different observers. They all refer to one substance, tungsten.

There are certain definite numerical relations between the frequencies of lines and absorption edges. These are apparently in all cases of the form :—

Difference between the frequencies of two absorption edges =frequency of a line.

Kossel drew attention some time ago to the probable existence of relations between the frequencies, but supposed that they involved line frequencies only. Duane and his collaborators find that their accurate expression brings in the

Spectrum Lines.

K Series.

	Siegbahn $CaCo_3=3{\cdot}029$	Duane $CaCO_3=3{\cdot}028$	Dershem $NaCl=2{\cdot}814$	Ledoux-Labard & Dauvillier.
$K\alpha_2$	0·21352	0·21341	0·2124	0·2128
$K\alpha_1$	0·20885	0·20860	0·2076	0·2053
$K\beta$	0·18436	0·18420	0·1834	0·1826
$K\gamma(\beta_2)$	0·17940	0·12901	0·1784	0·1768

L Series.

Intensity (Siegbahn)	Siegbahn $NaCl=2{\cdot}81400$ $CaCO_3=3{\cdot}02904$	Overn NaCl =2·814	Dershem NaCl=2·814	Compton $CaCO_3=3{\cdot}0279$
3 λ......	1·67505	...	...	...
3 α_2 ...	1·48452	1·4839	1·4828	1·4846
10 α_1 ...	1·47348	1·4731	1·4722	1·4736
η ...	1·417	1·4177	1·4163	...
2 β_4 ...	1·29874	1·2984	1·2977	1·2987
...	1·2871	1·2872	1·2868	...
8 β ...	1·27917	1·2793	1·2784	1·2792
3 β_3 ...	1·26000	1·2598	1·2586	1·2602
6 β_2 ...	1·24191	1·2434	1·2416	1·2421
...	1·2395	1·2355	...	...
...	1·2205	1·2212	1·2202	1·2187
...	1·2118	1·2132	...	...
...	...	1·2097	1·2098	...
β_5 ...	1·2031	1·2021	1·1773	...
...	1·1284	1·1302	1·1292	...
3 γ_1 ...	1·09553	1·0967	1·0953	1·0965
...	...	1·0794	...	...
...	...	1·0724	1·0705	...
1 γ_2 ...	1·06584	1·0659	1·0648	1·0653
1 γ_3 ...	1·05965	1·0596	1·0587	1·0584
...	...	1·0446	1·0427	...
γ_4 ...	1·02647	1·0263	1·0253	1·0251

absorption edges. For example, in a recent Paper (" Physical Review," July, 1919, p. 67, April, 1920, p. 329), he gives the following figures :—

Tungsten.

K Spectrum.

	Wave-length.	Reciprocal of wave-length.
$K\alpha_2$	0·21341	4·6858
$K\alpha_1$	0·20860	4·7938
$K\beta$	0·18420	5·4290
$K\gamma$	0·17901	5·5864
Absorption edge *Ka*	0·17809	5·6151

There are at least two absorption edges in the L series, given by Duane as $La_1=0{\cdot}8240$ and $La_2=0{\cdot}9323$.

$Ka-La_1$ (the difference between the reciprocals of two absorption edges)

$=5{\cdot}6151-0{\cdot}8240$

$=4{\cdot}7911$

$=Ka_1$ within errors of experiment,

$Ka-La_2=5{\cdot}6151-0{\cdot}9323$

$=4{\cdot}6828$

$=Ka_2$ within errors of experiment.

Also, Stenström has observed three absorption edges in the M series of uranium and thorium, from which the reciprocal for tungsten can be found by extrapolation; these are :—

$$Ma_1=0{\cdot}152,\ Ma_2=0{\cdot}157,\ Ma_3=0{\cdot}182.$$

The difference $Ka-K\beta=5{\cdot}6151-5{\cdot}4290$

$=0{\cdot}1861$,

and again the agreement is within the experimental errors.

The K series of lines peculiar to any substance can be excited either by X-rays, or by electrons. In the former case the incident X-rays must have a frequency greater than that of the absorption edge; in other words, the quantum energy of the exciting X-ray must be greater than the quantum energy of the edge. In the latter case, the energy of the electron must exceed the same limit. The whole series is excited at once, and the intensities of the various numbers appear to be always in the same proportion. If in the X-ray bulb the speed of the whole electron stream is increased, the intensity of the series increases with it. Papers dealing with this branch of the subject are by Hull ("Physical Review," January, 1916), Webster ("Physical Review," June, 1916), Webster and Clark ("Proceedings" Natural Academy of Science, 1917, p. 181), and Blake and Duane ("Physical Review," December, 1917).

It is reasonable to suppose that any one electron cannot excite more than one member of a series since its energy is not greater than the quantum energies of any two lines. In that case the relative strength of the lines must represent the relative chances of those lines being excited by any one electron. This is in agreement with the fact pointed out by

Duane that the well-marked absorption edge La_1 goes with the strong line $K\alpha_1$, and the weaker La_2 with the weaker line $K\alpha_2$.

An electron can excite X-rays of all frequencies for which the quantum energy is less than its own energy; so that a stream of cathode rays excites a continuous spectrum limited at the high-frequency end. But it does not excite any of the series of lines unless, as explained above, its energy exceeds the quantum energy of the edge belonging to those lines.

To each edge in the L series belongs its own set of lines. There are certain differences of character between set and set. The frequencies of the lines α_2, α_1, β_2 and β_5 which belong to La_1 increase linearly with the atomic number of the atom from which they come; while the frequencies of the lines $\eta\beta_4\beta_1\gamma_1\gamma_2$ which belong to La_2 increase rather more rapidly. (*See* Webster, "Physical Review," March, 1920, p. 238.)

The frequency of the absorption edge, perhaps the most singular of all the frequencies associated with an atom, increases steadily with the atomic number, following Moseley's principle. Its importance has led to its being carefully measured by de Broglie, Duane, Siegbahn, Wagner and others; and it is clearly shown that the ratio of frequency to atomic number though constant for the smaller numbers rises somewhat sharply for the larger. Duane puts forward an interesting view that the *velocity* of the electron associated with the absorption edge rises linearly with atomic number, and that the uneven rise of the *energy* (energy of electron or quantum energy of exciting X-ray) is due to the increase of the mass of the electron with its speed.

The relation between the frequencies of lines and edges in the X-ray spectra have been closely examined because of the bearing which they have on the theories of Sommerfeld.

For although Sommerfeld's main discussion refers to the origin of the fine structure of the ordinary spectrum lines in the cases of helium and hydrogen, yet he has also considered the case of the very high frequencies of the X-rays. The frequencies of X-rays are of the order of a thousand times as great as those of visible rays, the electron speeds which excite and are excited by them are so large that the movements of the electrons are readily detected and measured; and, generally, the whole phenomena depending on the connection between wave motion and electron movement are displayed on a far more open scale than in the case of light. At the

same time, they are much less complicated. Moreover, and again as a consequence of high electron speed, the effects of the varying mass of the electron are much more obvious. Thus the study of X-ray spectra, now that measurements can be made with so much precision, may be expected to make a contribution of special value to the radiation question.

Mr. CHARLES DARWIN gave the following account of the theory underlying the production of X-ray spectra: We are to consider the application of Bohr's theory to X-rays, and especially the work of Sommerfeld in this country. To recapitulate the theory, the parts of an atom move according to the laws of dynamics, but not all dynamical motions are permissible. There is a series of permissible states in which the atom can exist without radiating, the elements of the orbits being determined by quantum principles. Radiation occurs by the system passing with loss of energy from one permissible state to another, according to the well-known law $W_2 - W_1 = h\nu$, to determine ν. Without any more definite idea of the structure we can get out a good deal about X-rays. First, Bohr's idea leads at once to the Ritz principle of combination, by which the sum or difference of frequencies is also an observed frequency. This is, in fact, observed in X-ray spectra, and hence the study of them can be brought down from that of the lines themselves to the energies of the initial and final states of each. It is convenient to have a word for these energies, and they are called levels. By the general study of the X-ray spectra we find that there is one and only one lowest level in the K region—*i.e.*, W_κ has a much larger $-ve$ value than for any other permissible state. In the L region there are two fairly near together, W_{L} and W_{L}' and a third and fourth (hitherto unexplained) called W_Λ and W_Λ'. The next group of levels is called M, and so on. They increase rapidly in complication. Then we can obtain radiations of frequencies given by $\nu(K_\alpha) = \frac{W_{\mathrm{L}} - W_\kappa}{h}$; $\nu(K_\alpha') = \frac{W_{\mathrm{L}}' - W_\kappa}{h}$ (a close doublet); $\nu(K_\beta) = \frac{W_{\mathrm{M}} - W_\kappa}{h}$; &c. Again, $\nu(L_\alpha) = \frac{W_{\mathrm{M}} - W_{\mathrm{L}}}{h}$; $\nu(L_\beta) = \frac{W_{\mathrm{M}} - W_{\mathrm{L}}'}{h}$, and so on. All this follows without any more precise assumption about the natures of the permissible orbits. A further support to our explanation is given by the

intensities. Suppose the level W_L is more likely to arise than W_L', and that $W_L > W_L'$. Then $\nu(K_\alpha)$ is stronger than $\nu(K_\alpha')$, and it is of higher frequency. But as W_L occurs with the — sign in the L series, we conclude that in this series the line of lower frequency will be the stronger. This is borne out by observation. So much for the general developments. To get any further, more definite assumptions must be made about the structure of the atom. We therefore imagine that it is made up of rings of electrons round a nucleus. We call the innermost ring K, the next L, and so on. Perhaps, K has 3 and L 9 electrons for the permissible position which has least energy. Suppose that a shock is given, say, by an incident X-ray. Nothing happens unless the frequency is such that $h\nu$ is energy sufficient to change the arrangement to another permissible position. If it is great enough absorption occurs. Now suppose we enquire what conditions are required to lift out a K electron. The L, M, &c., rings are full up, so it is no use trying to put it there. Therefore, the minimum frequency which can be absorbed corresponds to W_κ, the energy necessary to lift a K electron right out of the atom. When this has occurred the K ring is short by one electron, and it proceeds to replace it. This it may do by borrowing one from one of the other rings. Say it comes from the L ring. Then the subsequent energy will be W_L instead of W_κ, and so the emission will be K_α. If from the M ring it will be K_β. Thus the absorption takes place in one step, but the subsequent emission may be in several. This explains the fact that the absorption frequency is higher than the emission frequencies of the lines.

Dr. E. A. Owen referred to the four kinds of frequencies of vibrations associated with X-ray spectra. In the case of the K-series spectra there are (1) several emission frequencies, (2) one frequency such that when multiplied by Planck's constant the product is equal to the minimum of energy the electron in the X-ray tube must have to produce that emission series, (3) one critical absorption frequency, and (4) one critical ionisation frequency. Moseley found from the examination of the emission spectra of the elements that the square root of the frequencies was approximately a linear function of the atomic numbers of the elements emitting them, the heaviest elements showing a slight divergence from the linear relation. Duane, examining the critical absorption frequencies of the elements from manganese to lead, found a similar relation with a similar irregularity in the case of the heavier elements.

Calculating the velocity of the electron in the X-ray tube necessary to produce a given radiation from the quantum relation

$$\frac{\frac{1}{2}m_0v^2}{\sqrt{1-v^2/c^2}}=h\nu,$$

he observed that the relation $v=\nu_0(N-3/2)$, where $\nu_0=0{\cdot}00678c$ was a more accurate representation of the experimental results ; the greatest variation in the value of v given by this equation from that deduced from experimental observations amounted to a little over 1 per cent. The critical absorption frequency is found to exceed the frequency of the γ-line in the emission series by 0·2 or 0·3 per cent., and the critical ionisation frequency to be equal to the critical absorption frequency to within less than 0·1 per cent. In the L-series spectra three critical absorption frequencies are observed, and Duane finds that each α-line frequency is equal within experimental error, to the difference between the K-critical absorption frequency and one of the L-critical absorption frequencies. Assuming the same law to hold for all emission spectra it ought to be possible to deduce the critical absorption frequencies of other series.

Some experiments on the measurement of the relative intensities of the α and $\beta\gamma$ peaks reflected by different crystals were described. For crystals containing atoms which do not selectively absorb radiation incident upon them, the relative intensities of radiations of two nearly equal frequencies should remain constant. If, however, one of the radiations is selectively absorbed whilst the other is not, the relative intensities of the reflected beams will be greatly altered ; in fact, the measurement of the relative intensity of two radiations of nearly equal frequencies would serve as a method of determining the critical absorption frequencies of atoms comprising the crystal, by comparison with a suitable standard of reference.

The method of measuring the intensity of a peak was that employed by Bragg, by turning the crystal at a definite rate from one side of the maximum to the other. This agreed well with another method of measurement in which the ordinates of the peak were experimentally determined and plotted, and the area measured directly. The K-series lines of Palladium and Rhodium were employed and reflections measured from the five crystals $CaCO_3$, CaF_2, SiO_2, S, CSi. No regular change in the ratio of the intensities of the peaks with specific

gravity of crystal, was observed, but in both crystals containing calcium the ratio was higher than in the other cases. On the assumption that the crystal selectively absorbs the radiation, the high ratio for the crystals containing calcium could be explained if calcium emitted a radiation of wave-length somewhere in the neighbourhood of $0{\cdot}52 \times 10^{-8}$ cm. This would have to be a radiation of the J-series. At present there is no definite proof that this series exists, so that possibly the phenomenon might be due to another cause.

Dr. E. H. Rayner asked the order of magnitude of the quantities W_{κ}, &c.

The President replied that they were of the order 10^{-11} c.g.s. units.

Prof. Fortescue proposed a vote of thanks to the President and the other speakers for their able survey of the work that had been done in X-ray spectroscopy.

Prof. Rankine seconded, pointing out the value to teachers of these co-ordinated accounts of a particular branch of physics at which they might not be working themselves.

I. *Ionisation and Excitation of Radiation by Electron Impact in Helium.* *By* FREDERICK S. GOUCHER, *Ph. D.*

NOTE.—This work has been carried out with the aid of a grant from the Council of Scientific and Industrial Research during the months February to July, 1919, at the Carey-Foster Physical Laboratory, University College, London University.

Introduction.

THIS investigation was undertaken as a continuation of the experiments of Davis and Goucher (1) on the ionisation and excitation of radiation by electron impact in various gases.

The method there employed was a modification of that first used by Lenard for the direct determination of the ionisation potentials of gases; the modification being introduced for the purpose of determining whether or not the effect hitherto interpreted as resulting from ionisation was not due at least in part to a photo electric action on the metal parts of the apparatus, produced by radiation from the bombarded gas atoms. It had been applied to the case of mercury vapour, hydrogen and nitrogen showing that for these gases at least two critical potentials existed, the lowest one of which in each case corresponded to the production of radiation by the electron impacts on the gas atoms, while ionisation did not take place until a higher potential was reached.

Intermediate critical potentials were found which, along with these others, bore definite relations to the spectra of these gases as calculated by the quantum theory of radiation. Analogous effects have been shown for the case of a number of the metallic vapours, by other experimenters using a different method (2).

The application of the method to the case of helium seemed specially desirable both from the theoretical point of view and because indirect experimental evidence pointed to a higher ionisation potential for this gas than had been found by direct measurement hitherto.

It was with this end in view that the investigation was undertaken. Before its completion, however, Horton and Davies (3) published an account of experiments on helium in which this method with extensions was applied. These showed that two critical potentials did exist for this gas, the lower of which—20·4 volts—corresponded to the production of

radiation only, while the higher—25·6 volts—corresponded to the ionisation potential. For a full discussion of the theoretical significance of these results and a comparison with previous experimental evidence, reference should be made to their Paper. It was considered advisable to continue with the investigation, however, because it was proposed to introduce certain modifications into the method which would render it more sensitive, with special regard to the separation of the ionisation and radiation effects.

The results obtained were in good agreement with those of Horton and Davies with the exception that some ionisation was found to accompany the production of radiation at the lower critical potential. The effect of impurity was suspected, but this could not be established before the investigation was discontinued. A recent Paper by Horton and Bailey (4) gives evidence for this view and offers an explanation for the marked effect of minute traces of impurity. That it is a result of impurity is probably correct, though the exact part this impurity plays is not so clear. Other experimenters (5) using the valve method for the measurement of ionisation independent of radiation effects, find ionisation at the lower critical potential only in cases where the gas must be quite pure and the apparatus comparatively free from electrode gas, though Rentschler (6) reports experiments in which the method was used indicating that ionisation did not take place until the bombarding electrons passed through a field of 26 volts.

It seems desirable, therefore, to describe in detail the results obtained in the course of this investigation and to discuss them in the light of these recent experiments.

Description of the Apparatus.

The apparatus used in this experiment was in general similar to that used in the experiments on mercury vapour, hydrogen and nitrogen, and need not be described here in detail. Certain modifications were, however, introduced in the construction of the measuring vessel, and improvements made in the method of gas manipulation, which seem worthy of note.

The platinum thimble used as an equipotential surface electron source in the former experiments was done away with, both on account of the difficulty of freeing it of occluded gas, and also because the method of estimating the critical values of potential was to be a purely relative one in which

corrections for distribution of velocities due to temperature and to the potential drop in the filament were no longer necessary. The source of electrons employed was simply a loop of tungsten wire requiring currents of the order of 2 amperes to heat it, and of such a length that the drop of potential as measured between the filament leads outside the vessel did not exceed one volt. Also, the amount of metal was reduced to a minimum in order to facilitate the pumping out of the apparatus and render the evolution of gas during the course

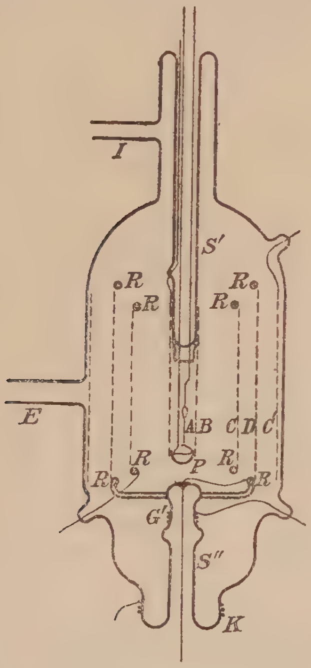


Fig 1

of measurement as small as possible. Instead, therefore, of using solid metal electrodes, so called "gas free," nickel wire gauze—mesh 20 to the inch—such as that used in the construction of wireless valves, was employed throughout. All supports were made of glass in so far as possible.

The cylindrical form of vessel was employed as before, with the filament along the axis.

Fig. 1 is a sectional view of the measuring vessel showing the arrangement of the essential parts. *A* represents the

tungsten loop used as the source of electrons, heated and supported by means of the leads sealed into the glass part S′S′ as shown. S' also served as a support for the gauze B and contained the seal of the lead to B as shown. The gauze B was bound to S' at its upper end by a fine metal wire, and to a glass cap P at its lower end, which served to insulate it from the filament support wire, which was so bent as to keep A concentric with B.

The glass part S'', to the enlarged parts of which glass cane frames were attached—the R's in the diagram—served as the support for the two cylindrical gauzes C and D. A fourth gauze C' was supported by the walls of the vessel as shown. The leads to C, D and C' were carried to the outside of the vessel through seals in the glass as represented.

D, being the collecting electrode and the one attached to the electrometer, had to be specially insulated, which was rendered easy by this method of construction. It was protected on the inside by the ring of platinum wire G', which was partially embedded in the glass and which could be earthed. On the outside, a piece of wire gauze was tightly bound about the glass part above the seal at K, which could also be earthed.

The outside diameter of the vessel was 4 inches, and the other dimensions are in proportion.

E represents the main exhaust tube, and I an inlet tube for the introduction of gas into the measuring vessel.

Fig. 2 is a diagramatic view of the vacuum system, and is of interest because of the simple means it affords of introducing, circulating, and recovering small quantities of gas, at the same time purifying it and making it possible to conveniently test for purity by means of an auxiliary discharge tube.

The system in general consisted of the measuring vessel V, and connections, including a liquid air trap T_1; a side tube leading to the main pumping system containing a mercury cut-off C; a side tube to McLeod gauge G; a small water cooled mercury condensation pump P_2 connected to the reservoir of a small Töpler pump R_1; connections to the gas reservoir R through the stopcock S_1; to a small Plücker tube D, and to the gas inlet tube I of the measuring vessel through the stop-cocks S_2 and S_3 and the charcoal tube T_2, as shown.

C_1, C_2 and C_3 represent mercury columns controlling the mercury cut off C, the Töpler pumps R_1 and for filling the

mercury reservoir R_2 used for operating the condensation pump P_2.

Having thoroughly exhausted the system, liquid air being kept on the traps T_1 and T_2, the cut off C could be closed and the required amount of gas admitted through S_1. By means of the condensation pump it was possible to circulate this gas through V, out through E, and the trap T_1, and in through the charcoal tube T_2 and the inlet I. Also by closing S_2 it was possible to readily remove the gas which could be quickly stored in R_1 at the low pressures used in these experiments. Or by closing S_3 the gas could be compressed into D, for purposes of spectroscopic examination.

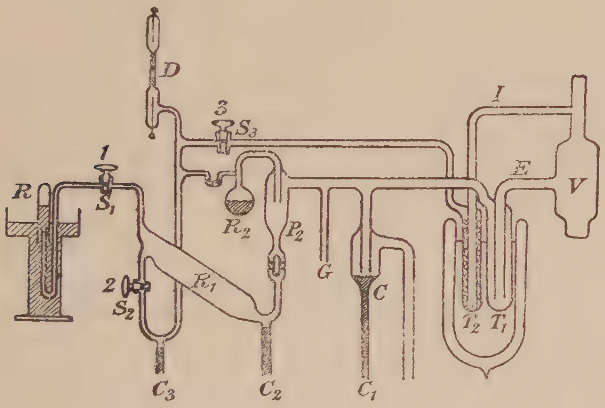


FIG. 2.

The main pumping system consisted of a large mercury condensation pump of the Langmuir type backed by a double Fleüss oil pump electrically driven.

The McLeod gauge was sensitive to ·00001 mm. of mercury.

The measuring vessel was furnished with an asbestos oven heated by means of a bunsen burner in which it could be baked out for many hours at a temperature over 300°C. During this heating the main pumping system could operate continuously and the filament be maintained at a high temperature throughout. This process could be kept up until the evolution of residual gases from the metal parts of the apparatus and the walls of the glass parts was reduced sufficiently to carry on the measurements.

The helium used in these experiments was very carefully purified by Professor Collie. The writer takes this opportunity to express his thanks to Professor Collie for this and other gases furnished for these experiments and for his kindly interest taken in the work.

The electrical apparatus used needs no special comment. Small storage cells were used as sources of potential. The photo-electric and ionisation currents were measured by means of an electrometer of the Dölezalek type. The electron currents were measured by means of a galvanometer.

Method of Measurement.

The method of making the measurements was in general the same as that used in the former experiments.

A field V_1 was applied between the negative end of the filament A and the gauze B in such a way as to accelerate electrons from A through B into the region $B\,C$, where they were retarded by a field V_2 applied between B and C. V_2 was kept at a constant value—usually about 4 volts greater than V_1—in order to prevent any of the original electrons from passing through C.

The gauzes C and C' were connected together throughout these experiments serving to control the field about the collecting gauze D. This field was varied by means of a potential V_3 which maintained CC' positive or negative with respect to D. In practice D was always kept at a potential zero, the rest of the system being controlled accordingly. This was done because of the electrometer used in measuring the current to D.

Curves could then be obtained showing the variation of current to D with variation of the accelerating field V_1 under different conditions of V_3. These curves furnish a means of estimating the critical potentials of the gas and of differentiating between the current effects due to actual ionisation in the gas and those due to a photo-electric action of the radiation from the bombarded gas atoms.

With V_3 small as compared with V_1 and V_2 positive ions when formed would reach D whether V_3 aided or opposed their motion, so that when V_1 exceeds the ionising potential, the current to D will be increasingly positive with increase of V_1 due to this cause alone, the tendency to increase occurring at the values of V_1 corresponding to the critical potential at which ionisation takes place. If, on the other hand, no posi-

tive ions were formed, and if all or part of the energy of the impacting electron—when exceeding a definite critical value—were transformed into radiation emitted from the bombarded atom, this radiation would fall on C, D, and C' and a photo-electric current would take place either from C and C' to D, or from D to C and C', depending on the direction of V_3. In other words D would charge with either an increasing negative, or an increasing positive current with values of V_1 progressively greater than the critical value at which the effect begins.

Both of these causes may, of course, be operative at the same time, but the interpretation of the curves rests entirely on these assumptions, discontinuities corresponding to critical potentials and the slope of the curves beyond these indicating the predominating effect.

Corrections, of course, had to be applied in the estimation of the absolute values of the critical potentials for which special measurements had to be made. This follows because it was not possible for all the electrons which were accelerated by the field V_1 to have an energy corresponding to this field, both on account of their initial velocity due to temperature, and on account of the drop of potential along the filament. Also, contact differences of potential are likely to occur between the filament and accelerating gauze.

In these experiments on helium it was proposed to measure only the relative values of critical potential and compare them with the ionising potential of mercury as measured in the same apparatus and under similar conditions, thereby eliminating the necessity for making these auxiliary measurements.

The method consisted in making a set of curves first in mercury vapour, then in helium under as nearly similar conditions as possible.

The mercury vapour was admitted to the measuring vessel by the removal of the liquid air from the traps and thoroughly pumping out by means of the main pumping system until the residual-gas evolution was reduced to an amount too small to register on the McLeod gauge. The pump was kept running during these measurements.

The mercury vapour could be removed sufficiently from the measuring vessel, so that it at least contributed no appreciable amount to the ionisation currents. This was done by means of circulating pure helium through the system

with liquid air on the traps for a short time, usually less than an hour being required.

For the purpose of making the measurements under similar conditions of electron emission in the two gases it was found necessary to insert a galvanometer in the battery circuit supplying the potential between A and B. For with equal filament current the emission was found to be quite different in the two cases, due to the effect of the gas, even when the potential V_1 was below the ionising potential of either gas. The filament current was therefore adjusted so as to give equal

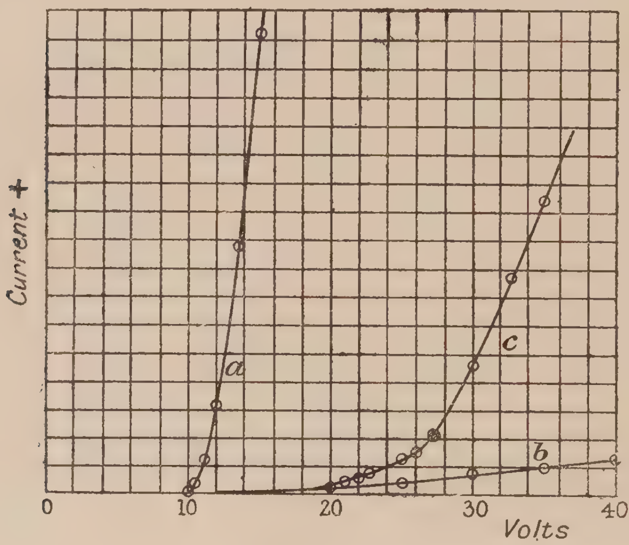


Fig. 3

electron currents in the two cases at a given value of V_1 below the ionising potential of mercury vapour.

Experimental Results.

Fig. 3 shows a typical set of curves obtained in mercury vapour and helium for the case where V_3 is zero. This is the case where the effect of radiation on the collecting electrode should be zero provided the state of field about D is actually zero.

The presence of the large field V_2 would effect the state of field between C and D to come extent, and this might be sufficient to cause a slight tendency for D to charge positively

in the presence of radiation even though no ionisation were present.

Curve (*a*) was obtained with mercury vapour present in the measuring vessel at a pressure corresponding to the temperature of the vessel, which in this case was slightly above room temperature. This should give a pressure of the order of 0·001 mm. The electron emission was adjusted so as to give a convenient reading on the electrometer over a range of a few volts above the ionising potential.

This curve shows a critical potential for mercury vapour at a value between 10 and 10·5 volts, which is in very good agreement with experimental values already obtained and with the value calculated by means of the quantum theory from the wave-length of the line 1188 Angstrom units, which is the head of one of the combination series for mercury vapour. This calculated value is about 10·4 volts, and probably represents the true ionising potential of mercury vapour.

We may then fairly assume that the great majority of the bombarding electrons in this apparatus, working under these conditions, have velocities, the energy of which corresponds to the values of field applied and measured as V_1.

Curve (*b*) was obtained after the removal of the mercury vapour from the measuring vessel. This was done by washing for some time with purified helium, liquid air being kept on both traps and the auxiliary condensation pump being kept running. This helium was then pumped out and the vessel exhausted by the main pumping system for several hours. Then, with this pump running, the measurements were made which are represented in the curve (*b*). That this positive current was not due to the ionisation of residual gas was considered likely, because further pumping did not cause it to diminish, and the pressure was too low to be recorded on the McLeod gauge. It was considered as probably due to the emission of positive ions from the gauze *B* under the bombardment by electrons from *A*, as has been pointed out by Horton and Davies, who obtained a similar "no gas" curve.

Curve (*c*) was obtained with purified helium in the measuring vessel at a pressure less than 0·01 mm. It is typical of a large number obtained at these pressures, and shows definitely two critical potentials, one about 20 volts where it breaks away from the "no gas" curve and one at about 26 volts.

This would indicate two critical ionising potentials, since in this case the field V_3 was adjusted so as to render the collecting electrode comparatively insensitive to the photo-electric effect. This condition may not be exactly fulfilled, as has been pointed out.

Fig. 4 shows curves obtained with variation of the field V_3. from $+1{\cdot}3$ volts to $-1{\cdot}3$ volts. (*a*) is the "no gas" curve, which does not change with the reversal of V_3, (*b*) the curve obtained when V_3 is such as to cause *D* to charge negatively under the action of radiation alone, viz., $V_3 - 1{\cdot}3$ volts;

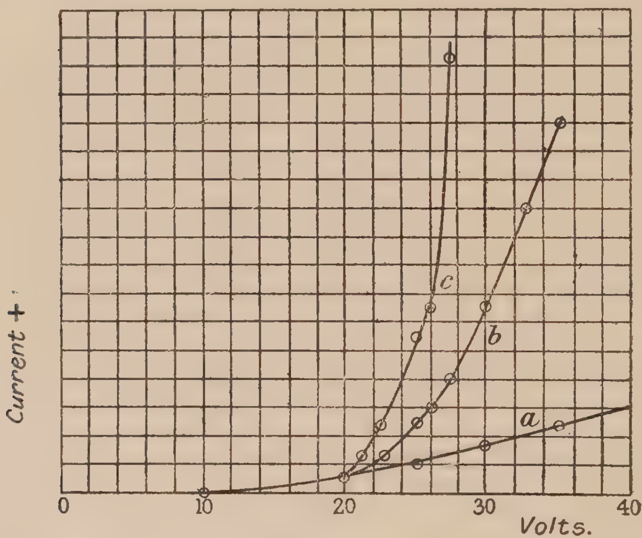


FIG. 4.

(*c*) the curve obtained when V_3 is such as to cause *D* to charge positively due to radiation alone, viz., $V_3 + 1{\cdot}3$ volts.

Since (*b*) as well as (*c*) have a positive tendency above the lower as well as the higher critical voltage, it would appear that under these conditions ionisation did occur at both the critical potentials, and that if radiation did take place the ionisation effect predominated.

Fig. 5 shows curves obtained with a higher pressure of helium—viz., 0·06 mm. V_3 was given the same value as before and reversed to test for radiation. Curve (*a*) was obtained with $V_3 + 1{\cdot}3$ volts, curve (*b*) with $V_3 - 1{\cdot}3$ volts. In this

case the radiation effect predominated and showed that strong radiation occurred at the lower critical potential, viz., about 20 volts. The slight tendency to a more positive slope of the curve above 26 volts indicates the increase of ionisation at that point.

The Effect of Radiation Alone on the Metal Parts.

In order to test the assumption that the positive slope of the curve (*b*), Fig. 4, above 20 volts, proves the presence of ionisa-

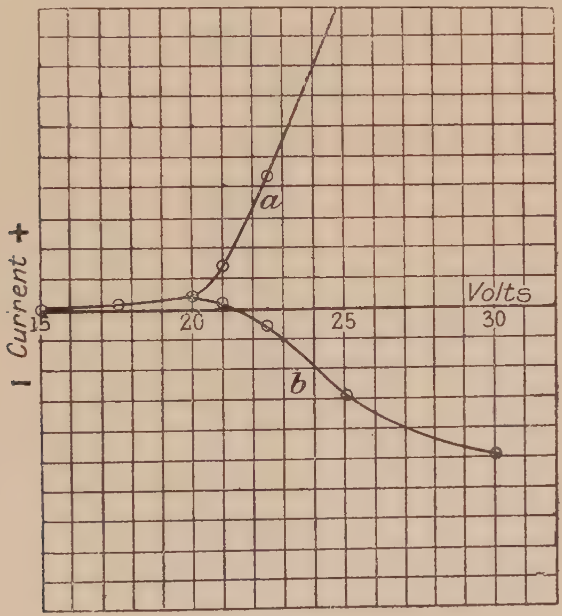


FIG. 5.

tion, and would have been negative if radiation alone were responsible for the charging up of *D*, the following modification was made in the measuring vessel.

The vessel in its modified form is shown in Fig. 6. Instead of the filament source of electrons, a source of ultra-violet radiation was substituted. This was done by means of a silica tube, *Q*, closed at its lower end as shown, containing a pool of mercury, *M*, above which was supported an oxide coated platinum filament. The electrons from the filament

could be utilised to produce a luminous discharge in mercury vapour, using the mercury pool as an anode. The silica tube was of the same diameter as the cylindrical gauze *B*, Fig. 1, and served as its support in the modified form, so that the geometrical arrangements of the gauzes remained the same.

The silica tube was sealed into the measuring vessel by means of the glass parts and wax joints, as shown, and was connected with a separate exhaust tube, *I′*.

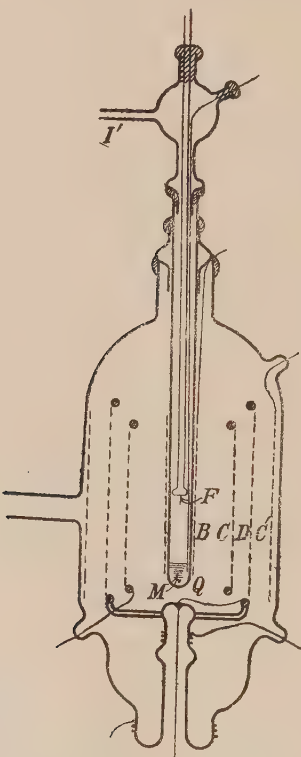


FIG. 6.

The exhaust tube *I′* was not connected to *I*, Fig. 2, as in the first part of the experiment; but directly to the large pump system beyond the mercury cut-off (*c*). *I* was sealed off, otherwise the vacuum system was the same.

The volumes both inside and outside *Q* with the cut-off open could be thoroughly exhausted. The cut-off could then

be closed, and gas to any desired pressure could be then introduced into the volume outside of Q. The pressure inside Q could be maintained at a constant low value as regards gases other than mercury vapour by keeping the pump running so that when temperature equilibrium was attained after heating the filament the pressure of mercury vapour remained quite constant.

The arc could easily be struck by applying a high potential between F and M and then maintained by a 25-volt field under the conditions here obtaining. The intensity of the discharge, and hence of the radiation emitted, could be controlled by the electron emission from the cathode.

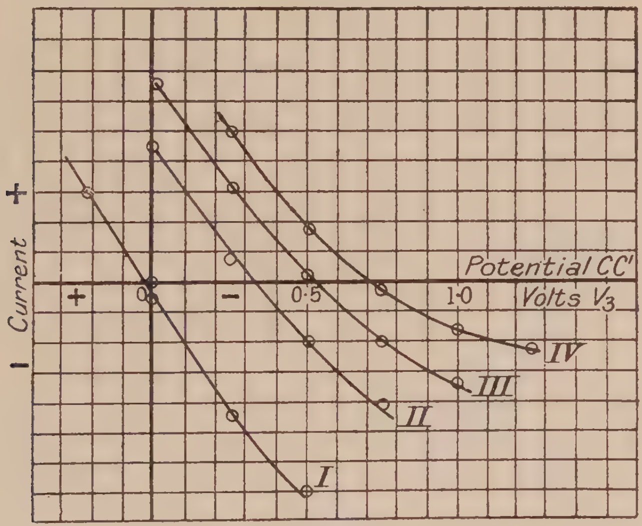


Fig. 7.

B was kept at the same potential as M, and was used to control the field V_2. So that it was possible to duplicate all the field conditions obtaining in the measurements made in the first part of the experiment, with the excption of the field V_1, which in this case was fixed at 25 volts; but this is of the same order of magnitude as the value of V_1 obtaining at the portion of the curves 20-26 volts with which we are specially concerned. In any case, the effect of V_1 on the field about D would be so small that this difference in the field conditions can well be neglected, as compared with the effect of V_2.

The following measurements were made: The arc was struck and maintained by a 25-volt field. Then, with V_2 fixed at a series of values ranging from zero to the highest values obtaining in the measurements made in helium in the first part of the experiment, a curve was obtained showing the variation of current to D with a variation of V_3 for each value of V_2. Fig. 7 shows such a set of curves obtained when the gas pressure in the measuring vessel was so low that there could be no appreciable ionisation. The cut-off C was kept open, so that this low pressure could easily be maintained. In these curves, *I.*, *II.*, *III.* and *IV.*, V_2 had the values 0, 6, 12

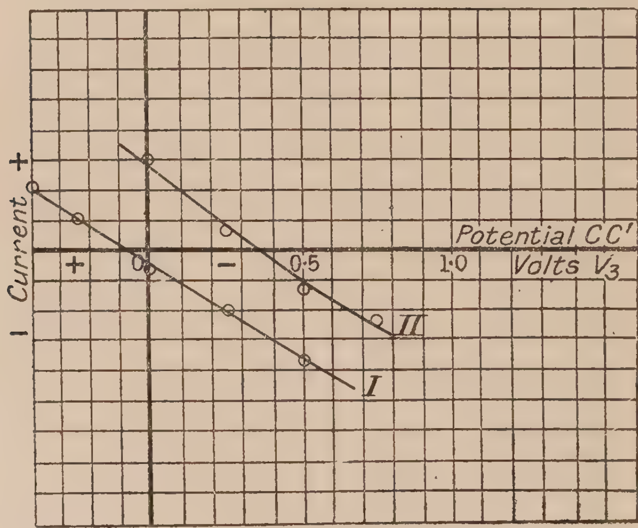


FIG. 8.

and 24 volts respectively, while V_3 was varied over the ranges shown, sufficient in each case to give a curve cutting the V_3 axis. The electrometer sensitivity was the same as that used in measuring the ionisation and photo-electric currents in the first part of the experiment, and the intensity of the mercury vapour discharge was adjusted to give photo-electric currents of the same order of magnitude.

These curves show that the assumption that V_3 controlled the field about D does not hold entirely, otherwise all of the curves should cut the axis at the point $V_3=0$. The shift, which is in the direction expected as a result of the field V_2,

is never more than 0·7 volt when V_2 has its maximum value. The slight shift of 0·1 volt to the left when $V_2=0$ shows that V_1 is here effective to this extent; but this would not be effective in the original measurements, as V_1 is there variable, and would be approximately zero when V_2 was zero.

In order to show the effect of helium on these radiation effects, a set of curves was obtained, Fig. 8, with helium present in the measuring vessel at a pressure corresponding to that used when the curves, Fig. 4, were obtained. Here *I* was obtained when $V_2=0$ volts, *II.* when $V_2=30$ volts. It is seen that this amount of helium diminished rather than increased the effect of the field V_2 on the photo-electric currents.

These curves, therefore, prove that with the value of V_3 used in making the original measurements in helium, if *D* charges positively it must be attributed to a predominance of ionisation currents, and could in no case be attributed to radiation, for with this value—1·3 volts—*D* would certainly charge negatively under the action of radiation alone.

For the purpose of further testing the apparatus as used in these experiments, curves were obtained both in mercury vapour and hydrogen. The results obtained agreed will with those obtained in the experiments of Davis and Goucher, except that in the case of hydrogen no ionisation was obtained below 16 volts.

Discussion of Experimental Results.

The conclusions arrived at in these experiments agree well with those obtained by Horton and Davies with the exception of the ionisation shown to accompany the production of radiation in the helium at 20 volts.

That this is due to the presence of a trace of gas having a lower ionisation potential than helium as pointed out by Horton and Bailey is probably correct. For even though this gas contributed no measurable ionisation current below the 20-volt critical potential due to direct electron impacts, there might still be enough to contribute a measurable current above this value due to radiation from the radiating helium atoms. This would be true both because the volume of gas from which positive ions could be drawn to the collecting electrode *D* would be larger when depending on radiation as the means of ionisation than it would when depending on electron impact, and also because a higher percentage of the

gas ions could be ionised by the radiation than by electron impact at the low pressures obtaining.

That the impurity causing this effect is mercury vapour which has not been removed from the measuring vessel seems very unlikely, both because of the washing out with pure gas passing through liquid air, and also because in the observations made on pure hydrogen no ionisation was observed below 16 volts. If there were sufficient mercury vapour present to contribute a measurable ionisation current it would have done so below 16 volts in the presence of the radiation which is known to take place at about 11 volts in hydrogen. The impurity is more likely to be electrode gas or gas coming off of the glass walls of the apparatus than any impurity present in the helium, because this was very carefully purified.

The gases coming off of the glass and metal parts of the apparatus are known to be mainly hydrogen carbon monoxide and water vapour. As charcoal immersed in liquid air was used in the gas circulating system, the hydrogen would be the most likely gas to accumulate in the circulating gas and cause the effect measured. If this is the case, part of the positive current in the "no gas" curve (*b*), Fig. 3, may be due to a steady evolution of hydrogen sufficient to maintain a slight pressure, even though the main pumping system was in operation. In this case the slight increase of positive current observed above the 20-volt critical potential could easily be accounted for by the additional positive ions produced by the radiation.

If the production of positive ions at the lower critical potential in helium could be shown not to be due to the presence of impurity—which was found to be impossible in this investigation without the construction of new apparatus —it would indicate the likelihood that some of the normal helium atoms were absorbing sufficient radiant energy, so that they could be ionised with a 20-volt electron impact.

Summary.

Measurements have been made of the critical potentials for helium by the method used in the experiments of Davis and Goucher and compared with the ionising potential of mercury vapour taken as a standard.

Assuming the ionising potential of mercury to be 10·4 volts, two critical potentials occur in helium. One at about 20 volts

and the other at about 26 volts. These critical values agree well with those obtained by Horton and Davies.

The effect of radiation alone on the metal parts of the apparatus was studied under conditions which would yield evidence of use in the interpretation of the results obtained when the production of both ionisation and radiation was taking place simultaneously.

The conclusion was reached that the lower critical potential was a radiation potential, though some ionisation was produced also at this potential. This, however, was attributed to the presence of impurity, probably hydrogen. The higher critical potential was that at which ionisation took place.

The writer wishes to thank Sir William Bragg for the apparatus and facilities used in the carrying out of this investigation and for his kindly interest in the experiments.

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DISCUSSION.

Mr. B. S. Gossling, speaking on behalf of himself and Mr. J. W. Ryde, said: Dr. Goucher has given a most illuminating critical discussion of his results. Material furthering this discussion may perhaps be found in some observations made by us last year in the laboratories of the G. E. C. bearing on the question of the existence and effect of impurities.

The object of the experiment was actually to trace the variation with time of the effects of a small quantity of impurity known to be present at the beginning. The method of observation was that described in a recent Paper by G. Stead and B. S. Gossling, ("Phil. Mag.," 1920, v. 40, p. 413), which, in the light of these later results requires qualification in respect of the statement that the ionisation potential of helium is 21 volts.

The invariable final result shown in the cases I have to bring forward, and in the very similar case described by Horton and Bailey ("Phil. Mag.," 1920,

v. 40, p. 440), is that no positive ions appear until voltages considerably higher than 20 or 21 volts are reached. This indicates that the lower ionisation point is not a characteristic of pure helium, but is due in some way to an admixture removable by the discharge.

The observations were made on about a dozen valves, all of which behaved in a generally similar manner. These were subjected to a more severe exhaust with heating by electronic bombardment lasting an hour or more on a better pumping system than that used by Stead in the earlier case, but after filling with helium to 0.7 mm. pressure were generally sealed off instead of being left over charcoal and liquid air.

Taking the current-voltage curves for a typical case and plotting the logarithms of current and voltage so that the " no gas " curve is very nearly straight, and the curvature at the various voltage values where ionisation sets in correspondingly more distinct, we find a pronounced breaking away from the " no-gas " curve at about 15.5 volts, indicating the presence of a very considerable amount of impurity, presumably hydrogen or carbon monoxide, or both. After half an hour's running, however, with an anode potential of 150 volts there is a very marked change, the curve obtained being like Stead's with the critical point rather above 21 volts. But later curves taken after seven, nine and twelve hours show a further, but much slower, progressive change, and finally no positive ions are found to appear before 25 volts at the least. In some of the sealed valves the final critical voltage was well above this value, and there were two cases where the valve was left over liquid air where the first definite curvature is at 27 volts. In a few sealed valves curves were obtained showing, like Dr. Goucher's, two critical points, at 20-21 volts and 24-25 volts.

What was happening seems to be that with the exhaust treatment given the valve was left in such a state that the impurities disappeared under the action of the discharge just as they would have done had the helium not been there.

Pumping with the electrodes cold will not remove the last traces of impurity, nor will charcoal, as Dr. Goucher and Mr. Stead agree. Very possibly the impurity is " condensed " on the surfaces of the electrodes, and is ionised in situ. The use of gauze electrodes of large total surface would favour this. A given quantity so condensed would give many more ions than the same amount spread throughout the tube.

On the other hand, prolonged passage of the discharge does in the first put a stop to the formation of positive ions between 15 and 20 volts, apparently by removing the impurity, even in a sealed tube, to a place where it is no longer ionised by impact. Above 20 volts the impurity may still be accessible to helium radiation for a time, but ultimately its effects disappear and we are left with an ionisation at between 25 and 27 volts of which there is a better probability that it is really due to helium.

The considerations here put forward seem to apply to the recent work of Compton (" Phil. Mag.," 1920, v. 40, p. 553) equally with that of Dr. Goucher.

The PRESIDENT asked if argon behaved in a similar way to helium as regards the effect of impurities.

Mr. GOSSLING mentioned that Stead's values for argon were also low, but whether this was due to impurities or not he could not say.

The PRESIDENT asked if these critical potentials corresponded to the production of lines in the spectrum of the gas.

Dr. GOUCHER said the radiation potential did correspond to a line. In mercury, for instance, the radiation potential was 4.9 volts, which, as McLennan had shown, corresponded to the production of a single line in the mercury spectrum. The many-lined spectrum was produced when the potential was at or above the ionisation potential. The results obtained by Messrs. Gossling and Ryde portray in a most striking manner the effect of impurity on the discharge in helium above its radiation potential. They give no evi-

dence of any secondary ionisation of the helium produced by the absorption of radiation combined with electron impact such as Compton appears to have found. When the present investigation was being carried out this secondary ionisation was considered as a possible explanation for the positive currents observed to accompany the production of radiation between 20 and 26 volts, and an attempt was made to test for this by studying the current voltage curves for different conditions of pressure and electron current density. It was expected that with increasing pressure and electron current density the 26 volt discontinuity would tend to disappear, showing an increase in the relative amount of secondary to primary ionisation. This effect was observed, but was not considered conclusive evidence, as the effect of impurity would also be to obscure the 26 volt discontinuity.

II. *The Location of Interference Fringes.* *By* J. GUILD, *A.R.C.Sc., D.I.C., F.R.A.S. (Optics Department, National Physical Laboratory.)*

It is frequently of importance in making certain determinations by the aid of interference fringes to know precisely where the fringes are situated. The problem has been worked out by various writers,* but their methods of treatment always seem somewhat arbitrary, inasmuch as the physical conditions involved in the problem are not clearly emphasised, and the dependence of the results on the circumstances under which the fringes are observed is never pointed out. Unnecessary assumptions are usually made which confuse the reader, and tend to invest the problem with a certain degree of mystery. For instance in Michelson's treatment, which is probably the best known to English readers, the source is assumed to coincide with one of the surfaces of the film. The assumption is unnecessary, as the position of the source does not affect the phenomena under the conditions which Michelson investigates. The following method of treatment may therefore be worth recording.

Let 1, 1′ and 2, 2′ (Fig. 1) be the section of a thick plate, the surfaces of which are very nearly but not quite parallel. Let SNA be a ray which undergoes reflection from the front surface. Let $SN'B$ be a ray emanating from the same point of the source (the same electron actually) which after reflection at the back surface intersects NA in some point such as P. Since all rays emanating from the same electron have a constant phase relationship interference or reinforcement will take place at P. But other rays from the same point in the source will intersect NA at different distances from N, consequently interference phenomena exist at all distances from the plate. What then determines the place where they will be most distinctly visible ?

Suppose that the observer's eye is at E. The pupil subtends an appreciable angle at the point P; so it receives from P not only the rays PA and PB (which in reality are so close together that if drawn to scale they would coincide), but a conical pencil made up of similar pairs of rays which come to P

* *E.g.*, Michelson, "Phil. Mag." (5), 13, 236; Feussner, Winkelmann's Handbuch der Physik, VI., p. 958.

from adjacent parts of the source via adjacent parts of the plate.

Since the thickness of the plate is not uniform, the phase retardation for the component pairs of the pencil will vary, and the illumination at P, which is due to all the light which reaches the eye from that point, can neither be of maximum nor zero brightness. If the phase variation comprised within the pencil is a few half wavelengths, the illumination will be the same as if there were no interference whatever. The same

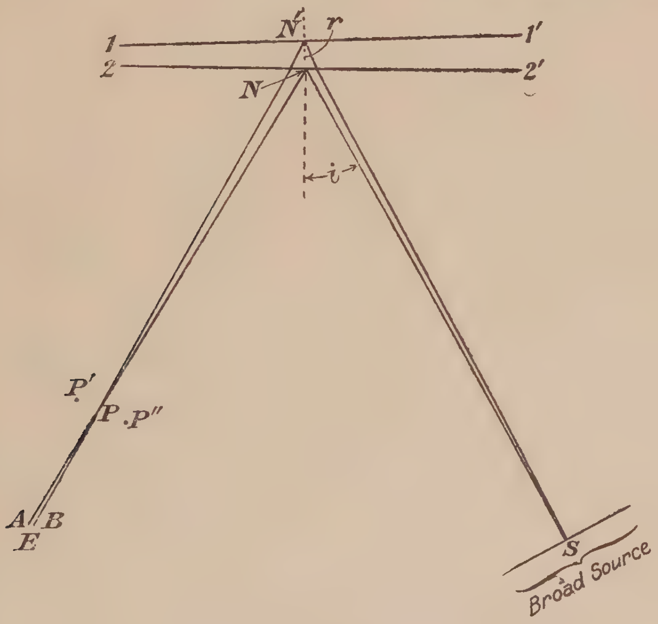


Fig. 1.

applies to other points in the field of view, such as P' and P'', so that the illumination is uniform and no fringes are visible. It so happens, however, that the rate at which the phase at P varies with the inclination of the rays which pass through it is not the same at all distances from the plate, and at a certain distance it has a minimum value. In this neighbourhood fringes will be seen if the cone of rays utilised from each point of the field of view is not too wide.

It is important to bear this last point in mind. Interference fringes are not entities like wires or focussed images, which

have a definite position in space. They have no existence, except in relation to the observing instrument. In general the disturbance in the ether is just as violent, and comprises as many phase components, at one point in space as another.* Visible fringes only indicate that the phase variation is very slight *within the particular bundles of rays which are utilised.* In some circumstances the slightest movement of the eye may cause a complete change in the character of the fringe system by slightly changing the position or direction of the bundles of rays employed for their observation. Also fringes may be distinctly visible to the eye, whereas an attempt to photograph them may entirely fail on account of the larger angle of the cone of light received by the camera lens.

To find where fringes are located, in the sense of the word indicated above, we may proceed as follows :—

Let the angle of incidence of the central ray of the cone, which reaches the eye from P (Fig. 1) be i_0 and let the thickness of the plate be t, and its refractive index μ. Let r_0 be the angle of refraction into the plate. Then the retardation for the axial pair is $2\mu t \cos r_0$,† where t is the thickness at N'.

The rays which reach the eye from P are comprised within a narrow pencil of circular section. Let us define any ray within this pencil by δ, its inclination to the axial ray, and φ, the azimuth of the plane containing the ray and the axis with respect to the plane of incidence. Let φ be zero when the angle of incidence of the ray has its maximum value, and 180° when it has its minimum value. All rays of inclination δ lie on a circular cone with its apex at P. This cone intersects the front surface of the plate in an ellipse of which the semi axis parallel to the plane of incidence is $x\delta/\cos i_0$, and the semi axis perpendicular to this plane is $x\delta$, where x is the distance PN. The distance from the centre to the point corresponding to azimuth φ is $\dfrac{x\delta}{\cos i_0}\cdot\dfrac{\cos\varphi}{\cos\psi}$, where $\tan\psi=\tan\varphi\cos i_0$. ψ is the inclination to the plane of incidence of the line joining

* Except when light from a point source is used, as described later.

† This relation is only strictly true when the two interfering rays are parallel both before and after incidence. Within the range of departure from this condition that may arise in practice, it is correct to a first approximation. Thus the retardation at P depends only on the thickness of the plate at the point where the ray is incident on the back surface, and on the angle of incidence on the back surface. It is independent of the distance of P or S from the plate.

the point of incidence of the axial ray to the point of incidence of the ray whose azimuth in the incident pencil was φ.

Thus the point of incidence of the ray δ, φ, is separated from the point of incidence of the axial ray by

$$\frac{x\delta}{\cos i_0} \cdot \frac{\cos \varphi}{\cos \psi}$$

in a direction making an angle ψ with the plane of incidence. If the angle between the surfaces of the plate is θ, and the line of greatest slope makes an angle α with the plane of incidence, the difference in thickness at the two points is clearly

$$\frac{x\delta}{\cos i_0} \cdot \frac{\cos \varphi}{\cos \psi} \cdot \theta \cos (\psi - \alpha),$$

θ being assumed positive when the azimuth of the direction of *increasing* thickness lies between 90° and —90°. Thus

$$\frac{dt}{d\delta} = \frac{x}{\cos i_0} \cdot \frac{\cos \varphi}{\cos \psi} \cdot \theta \cos (\psi - \alpha). \quad . \quad . \quad . \quad (1)$$

The angle of incidence of the ray δ, φ, is given by

$$\cos i = \cos i_0 \cos \delta - \sin i_0 \sin \delta \cos \varphi$$

(from the properties of spherical triangles). Where δ is very small, as in the case we are considering

$$\cos i = \cos i_0 - \delta \sin i_0 \cos \varphi.$$

The phase retardation between this ray and its partner from the back surface is, to a first approximation,

$$\begin{aligned} \rho &= 2\mu t \cos r = 2t\sqrt{\mu^2 - 1 + \cos^2 i} \\ &= 2t\sqrt{\mu^2 - 1 + \cos^2 i_0 - 2\delta \cos i_0 \sin i_0 \cos \varphi}, \\ &= 2\mu t \cos r_0 - 2t \frac{\delta \cos i_0 \sin i_0 \cos \varphi}{\mu \cos r_0}. \quad . \quad . \quad . \quad . \quad . \quad (2) \end{aligned}$$

From (1) and (2)—

$$\frac{d\rho}{d\delta} = -2t \frac{\cos i_0 \sin i_0 \cos \varphi}{\mu \cos r_0}$$
$$+2\left(\mu \cos r_0 - \frac{\delta \cos i_0 \sin i_0 \cos \varphi}{\mu \cos r_0}\right) \frac{x}{\cos i_0} \frac{\cos \varphi}{\cos \psi} \theta \cos (\psi - \alpha).$$

In the immediate neighbourhood of the axial ray (*i.e.*, $\delta = 0$), this will be zero, and the fringes will therefore be most distinct, when

$$x = \frac{t \sin r_0}{\mu \theta} \cdot \frac{\cos^2 i_0}{\cos^2 r_0} \cdot \frac{\cos \psi}{\cos (\psi - \alpha)}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From equation (3) it follows that x can only be independent of ψ, the condition that P should be the point of minimum confusion for rays in all diametral planes of the pencil entering the eye, if $a=0$—*i.e.*, if the plane of incidence is parallel to the direction of slope of the surfaces. In this case the fringes are perpendicular to the plane of incidence and the terms in ψ cancel out. As the direction of the fringes departs from this, the visibility falls off. Thus, with thick films at oblique incidence fringes can only be seen distinctly when their direction is perpendicular to the plane of incidence.

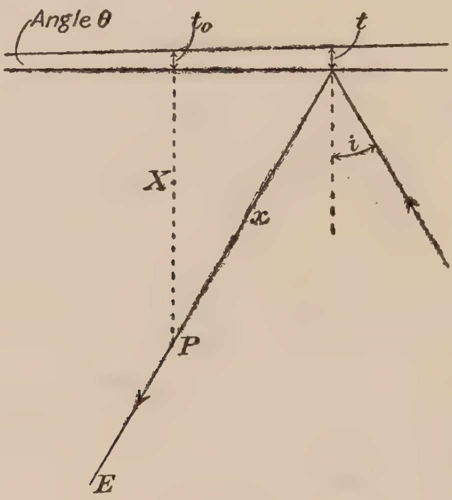


FIG. 2.

Most frequently in practice the medium between the surface is air, in which case

$$x=\frac{t \sin i_0}{\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\text{A})$$

In equating $\frac{d\rho}{d\delta}$ to zero we assumed δ to be negligibly small. The retardation is therefore only constant for a very small cone, and may vary quite appreciably towards the boundary of the pencil which enters the eye. It is therefore usually possible to sharpen up the fringes considerably by placing a small diaphragm in front of the pupil to restrict its angular aperture.

Equation (3A) is equivalent to that obtained by Michelson; but he expresses it somewhat differently. Michelson's formula is $X=\frac{t_0 \tan i}{\theta}$; where X is the *perpendicular* distance from the plate to the fringes and t_0 is the thickness of the plate *at the point where the perpendicular cuts it*. The equivalence will be clear from Fig. 2.

$$x=\frac{X}{\cos i}; \text{ also } t=t_0+X \tan i \,.\, \theta.$$

$$X=\frac{t_0 \tan i}{\theta}=\frac{t \tan i}{\theta}-X \tan^2 i,$$

or,

$$\frac{X}{\cos^2 i}=\frac{t \tan i}{\theta}.$$

$\therefore$

$$x=\frac{X}{\cos i}=\frac{t \sin i}{\theta}.$$

It is most convenient, in the author's estimation, to express the result in terms of the distance along the ray and the thickness at the point of incidence. These quantities always exist, and are measurable; but the quantities X and t_0 in Michelson's formula can usually only be obtained indirectly by calculation from x and t. We see from equations (3) or (3A) that in general the fringes lie in front of the plate when θ is positive on the convention adopted—*i.e.*, when the observer's eye is on the same side of the point of incidence as the line of intersection of the surfaces. If θ is negative they lie behind the plate. If θ is zero they are at infinity, and if either i or t is zero they are located at the plate. In the usual cases which arise in practice the value of θ is constant for all parts of the field of view. In viewing Newton's rings between a curved surface and a flat plate θ varies; but t is very small, so that the fringes are always at or close to the surface.

In cases where the interference is not produced at an actual film, but by an arrangement which is optically equivalent to a film, it is possible for such an equivalent film to have different angles for rays incident on it in different directions, and the location surface of the fringe system observable by the naked eye in such cases is affected accordingly.

Fringes in Parallel Light.—Fringes may be formed by a point source of light at infinity. In this case the rays meet every part of the plate at the same angle of incidence. In

order that the eye may receive light from more than one point of the plate at a time, the rays after reflection are received by a lens, in the focal plane of which they produce an image of the point source. The eye is placed at this point, and sees an area of the plate, equal to that of the lens, illuminated. Any fringes observed can only be due to variations in the optical thickness of the plate, since the angle of incidence is constant. They are, therefore, contours of film thickness. It is evident that the pairs of interfering rays from one point of the surface never intersect these from any other point;

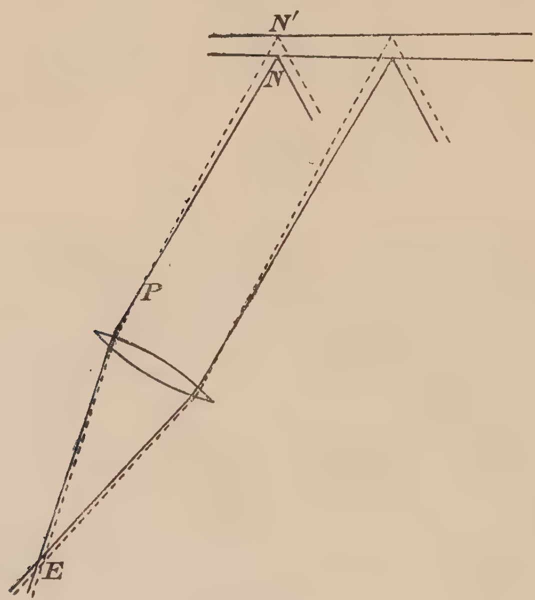


FIG. 3.

consequently the considerations which we applied in the case of a broad source do not apply at all in this case. From any point P, Fig. 3, only the rays from the points N and N' ultimately reach the eye at E.

Similarly, for any other point along the ray NP, the illumination is due simply to that ray, and to the ray from the rear surface which meets it in the point considered. The actual phase difference varies at different distances along NP, except when the film is parallel, but there will be no components of varying phase at any one point. Fringes are

therefore equally distinct at all distances from the plate, and will appear to be located either at the plate or at any aperture which bounds the field of view. Thus, in using fringes of this type for testing optical parts such as prisms and lenses by placing them in one beam of a Michelson interferometer, the fringes always appear coincident with the aperture of the system under test.

ABSTRACT.

The Paper treats of the conditions under which interference fringes, produced by reflection of light from the two surfaces of a "thick plate," are visible to an observer. The treatment lays stress on the physical significance of the term "location" as applied to interference fringes, and the dependence of the observed phenomena on the conditions of observation. For a broad source of light a formula is obtained which is equivalent to that derived by Michelson. For a point source of light at infinity, it is shown that the fringes obtainable are equally visible at all distances from the plate.

III. *Fringe Systems in Uncompensated Interferometers.* By J. Guild, *A.R.C.Sc., D.I.C., F.R.A.S.* (*Optics Dept., National Physical Laboratory.*)

In the investigations of the fringe systems obtained with interferometers, such as Michelson's for instance, in which the instrument is equivalent to an air film between two parallel reflecting surfaces, the instrument is always assumed to be accurately compensated, in the sense that for all possible rays the thickness of glass to be traversed in either of the two beams is identical. This is accomplished by the well-known means of providing a compensating plate of the same thickness and refractive index as the semi-silvered plate, placing it parallel to the latter in the beam which only passes once through it. The glass paths of the two beams are then exactly equivalent for all possible incident rays, and the interferometer is in effect reduced to a simple air-film, of thickness depending on the positions of the mirrors, the character of the fringes being readily deduced from the theory of thick plates of refractive index unity.

But in the recently developed method of testing optical parts by placing them in one of the beams of a large Michelson interferometer *, it is impossible to secure exact equality of glass paths, except in the simple case when parallel-sided slabs are under test. Even then it would require a separate compensator for every plate tested, and in practice the instrument is usually employed without any compensator at all.

As the interference phenomena in an uncompensated interferometer do not appear to have been described, an investigation of them may be of interest.

We shall assume, to commence with, that the mirrors have been adjusted to be at an equal distance from the semi-silvered surface, and that the surfaces of the equivalent film are perfectly parallel. The arrangement is really equivalent to a thin film, but the rays reflected from one of the surfaces pass through an oblique slab of glass before and after reflection, whereas the rays reflected from the other surface do not pass through the slab.

Let A, Fig. I, be a ray which does not pass through the slab and B a ray which does. The latter, in passing from

* F. Twyman, "Phil. Mag.", Jan. 1918, p. 49, and "Phot. Journ.", Nov., 1918, p. 239.

q to m, traverses a path the air-equivalent of which is $qn+\mu.nm$. The other ray, for which the slab is non-existent, traverses the air path pm. On account of its passage through the slab, therefore, the ray B suffers a retardation with respect to the ray A of $(qn+\mu\,.\,nm)-pm=\mu\,.\,mn-mo$

$$=\mu\,.\,mn-mn\cos\widehat{omn}.$$

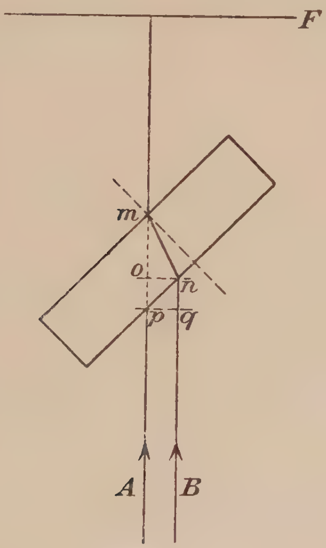


FIG. 1.

Thus if we denote the retardation by ρ, the thickness of the slab by t, and the angles of incidence and refraction by i and r respectively,

$$\rho=\mu\frac{t}{\cos r}-\frac{t}{\cos r}\cos(i-r)$$

$$=\mu\frac{t}{\cos r}-t\cos i-\frac{\mu t}{\cos r}\sin^2 r$$

$$=t(\mu\cos r-\cos i).$$

If these rays strike the surfaces of the film normally, they retrace their respective paths exactly and arrive back to the observer with a retardation 2ρ.

This retardation is clearly the same for all corresponding pairs of rays which meet the film normally; but it varies for pairs of rays which are inclined to the normal. It is only

necessary to consider the case of rays whose incidence on the film F is nearly normal; for, as we shall see, the fringes quickly become too close to be visible as the angle of incidence increases.

Consider then a pair of rays, not necessarily in the plane of the paper, inclined at a small angle δ to the pair AB. If i_0 is the angle of incidence on the oblique slab of the ray B, and i is the angle of incidence of the ray which is inclined to it by an angle δ, then, by a simple theorem in spherical trigonometry,

$$\cos i = \cos i_0 \cos \delta + \sin i_0 \sin \delta \cos \varphi,$$

where φ is the angle which the plane of the angle δ makes with the plane of the angle i_0, *i.e.*, with the plane of the paper in the figure. By considering all values of φ we consider all parts of a conical pencil coaxial with the rays A and B.

After reflection at the film, the rays will return inclined to the central ray by the same angle, but with its sign reversed, and will meet the oblique slab on their return journey with a new angle of incidence corresponding to the substitution of $-\delta$ for δ in the equation for $\cos i$.

Let ρ_1 be the retardation for the journey towards F.

Then $$\rho_1 = t(\mu \cos r_1 - \cos i_1)$$

whence $$2 \cos i_1 = (\mu^2 - 1)\, t/\rho_1 - \rho_1/t \quad . \quad . \quad . \quad . \quad . \quad (i)$$

and $$2 \mu \cos r_1 = (\mu^2 - 1)\, t/\rho_1 + \rho_1/t. \quad . \quad . \quad . \quad . \quad (ii)$$

Similarly for ρ_2 and i_2 the retardation and incidence angle on the return journey.

Also $$\cos i_1 = \cos i_0 \cos \delta + \sin i_0 \sin \delta \cos \varphi,$$

and $$\cos i_2 = \cos i_0 \cos \delta - \sin i_0 \sin \delta \cos \varphi.$$

From (i) and the sum and difference of $\cos i_2$ and $\cos i_1$ we get

$$\left[(\mu^2-1)\frac{t}{\rho_1\rho_2} + 1/t\right](\rho_2 - \rho_1) = 4 \sin i_0 \sin \delta \cos \varphi \quad . \quad (iii)$$

and

$$\left[(\mu^2-1)\frac{t}{\rho_1\rho_2} - 1/t\right](\rho_2 + \rho_1) = 4 \cos i_0 \cos \delta. \quad . \quad . \quad (iv)$$

We may expand ρ_1 and ρ_2 in a series of powers of $\sin \delta$, say

$$\rho_1 = \rho_0 - \alpha \sin \delta + \beta \sin^2\delta - \quad . \quad . \quad . \quad . \quad . \quad . \quad . \&c.$$

$$\rho_2 = \rho_0 + \alpha \sin \delta + \beta \sin^2\delta + \quad . \quad . \quad . \&c.$$

Substituting in (*iii*) and taking, as a first approximation,

$$\rho_1\rho_2=\rho_0^2,$$

$$2\sin i_0\cos\varphi=\alpha\left[(\mu^2-1)\frac{t}{\rho_0^2}+1/t\right]=\frac{2\alpha}{\rho_0}\cdot\mu\cos r_0,$$

whence $\alpha=\dfrac{\sin i_0\cos\varphi}{\mu\cos r_0}\rho_0.$

Similarly, by putting $\cos\delta=1-\frac{1}{2}\sin^2\delta$ and taking $\rho_1\rho_2$ to a second approximation, viz.,

$$\rho_1\rho_2=\rho_0^2-\alpha^2\sin^2\delta+2\beta\rho_0\sin^2\delta,$$

we obtain from (*iv*)

$$\cos i_0=\beta\left[\frac{(\mu^2-1)t}{\rho_0^2}+\frac{1}{t}\right]-\frac{\alpha^2(\mu^2-1)t}{\rho_0^3}.$$

From which

$$\beta=\frac{t}{2}\left[\cos i_0-\frac{\cos^2 i_0}{\mu\cos r_0}+\frac{(\mu^2-1)\sin^2 i_0\cos^2\varphi}{\mu_3\cos^3 r_0}\right]$$

$$+\text{terms in }\sin^4\delta,\ \&c.$$

Thus the difference between the retardation of the ray δ, φ, and the retardation of the normal ray ($\delta=0$) is

$$R=\rho_1+\rho_2-2\rho_0=2\beta\sin^2\delta$$

$$=t\sin^2\delta\left[\cos i_0-\frac{\cos^2 i_0}{\mu\cos r_0}+\cos^2\varphi\,\frac{\sin^2 i_0(\mu^2-1)}{(\mu\cos r_0)^3}\right]. \quad . \quad (1)$$

Hence, if the circumstances of the observation are such that different parts of the field of view correspond to rays of different inclinations, as, for instance, if the fringes are due to a broad source and are observed at infinity or in the focal plane of a lens (*i.e.*, the plane conjugate to infinity) a fringe system will be seen. The centre of the system will be the point in the field corresponding to normal incidence on the film; and on moving outwards from this point, a fringe will be encountered every time $\rho_1+\rho_2-2\rho_0$ increases by one wave-length. For any point distant δ (in angular measure) from the centre of the system, the excess of phase retardation over that at the centre, is

$$R=\rho_1+\rho_2-2\rho_0=t\sin^2\delta(A+B\cos^2\varphi)$$

in which A and B are essentially positive constants. Each fringe is the locus of points for which R is constant, so that δ, the (angular) radius vector of the fringe, is given by

$$\sin^2\delta,\text{ or simply }\delta^2,\ =R/t(A+B\cos^2\varphi). \quad . \quad (1a)$$

The fringes are, therefore, ellipses of which the principal axes are respectively parallel and perpendicular to the plane of incidence of the central ray of the system on the oblique slab. The minor axis is parallel to this plane and is $\{R/t(A+B)\}^{\frac{1}{2}}$. The major axis is $\{R/tA\}^{\frac{1}{2}}$.

We may regard these results indifferently as applying to a slab placed in one of the paths of an otherwise compensated interferometer, or to the half-silvered slab of an uncompensated interferometer. With an uncompensated interferometer, therefore, adjusted so that the mirrors are equally distant from the semi-reflecting surface, the fringes observed at infinity are ellipses distributed and shaped in accordance with 1(a). The system is clearly identical with the Newton's rings obtained between an astigmatic lens and a flat surface.

In the case of the half-silvered slab, i_0 is usually 45 deg., in which case the values of A and B are 0·334 and 0·266 respectively, assuming $\mu=1{\cdot}52$. In this case, therefore, the distances of points where the retardation differs by half a wavelength from that at the centre, *i.e.*, the distances of the black fringes if the centre happens to be bright, or vice-versa, are given by

$$\delta_h^2=\frac{(2n-1)\lambda/2}{0{\cdot}600t}$$

in the horizontal direction, and by

$$\delta_v^2=\frac{(2n-1)\lambda/2}{0{\cdot}334t}$$

in the vertical direction. The ellipticity of the fringes, as measured by the ratio of their axes, is $\sqrt{0{\cdot}600/0{\cdot}334}=1{\cdot}34$.

The horizontal distance to the first fringe $=\sqrt{\dfrac{\lambda/2}{0{\cdot}600t}}$,

which, for the green line of mercury, $=0{\cdot}0067/\sqrt{t}$ radians, $=23'/\sqrt{t}$. If the fringes are observed in the focal plane of a lens of focal length f, the linear distance is $0{\cdot}0067f/\sqrt{t}$.

Taking f as 30 cm. and $t=1{\cdot}8$ cm., which are common values in Hilger interferometers, the horizontal radius of the first fringe is 1·5 mm.

Effect of Additional Air Film.—The preceding treatment assumes that the distances of the two mirrors from the half-silvered surface are equal. In practice this is not necessarily so. In fact, it is not usually so. If the distances differ by x,

one of the rays has, in effect, to travel twice through a slab of air which the other ray has not. The central ray of the system passes through it normally, and the retardation due to it is $2x$. For rays inclined at an angle δ to the central ray, the retardation is $2x \cos \delta = 2x(1-\frac{1}{2}\delta^2) = 2x - x\delta^2$. If the extra air path is in the same side as the glass slab, the total retardation is the algebraic sum of that due to the slab and the extra air path :—

$$R = \delta^2 t(A + B \cos^2\varphi) - x\delta^2$$

$$= \delta^2 \{(A - x/t) + \mathrm{B} \cos^2\varphi\} \quad . \quad . \quad . \quad . \quad (2)$$

If the extra air path is in the other side, the total retardation is the difference of that due to the slab and that due to the extra air path, and

$$R = \delta^2 t \{(A + x/t) + B \cos^2\varphi\} \quad . \quad . \quad . \quad . \quad 2(\mathrm{a})$$

It is evident that the fringes will change in form as x varies. If there is a large excess of air path in the same side as the glass, the quantity $A - x/t$ in equation 2 will be large and negative. The fringes will be ellipses with their major axes horizontal. As the air path is diminished, the numerical value of $A - x/t$ decreases, and the fringes become of greater ellipticity until, when $A - x/t = -B$, they are horizontal straight lines, similar to the Newton's "rings" obtainable with a cylindrical lens and a flat plate. As x is diminished still further the fringes are hyperbolas, whose asymptotes rapidly open out from the horizontal axis and, after passing through the rectangular position when $A - x/t = -B/2$, close in on the vertical axis. When $A = x/t$, the asymptotes coincide with this axis and the fringes are vertical straight lines. A further reduction of x renders $A - x/t$ positive but small. The fringes are now elongated ellipses with major axes vertical. As $A - x/t$ increases, they crowd closer together and the ellipticity diminishes until, when $x = 0$, the condition is arrived at which we treated in the first instance.

If we continue to advance the mirror, which is equivalent to introducing an air film into the other beam, the effect is a continuous enlargement of $A + x/t$, so that the fringes crowd more closely together, and become less elliptical on account of the greater preponderance of the term which is independent of φ. A particular case of practical interest is when x is approximately equal to ρ, the retardation of the oblique slab. When this is so the total optical paths in the two sides of the

interferometer are equal, and the fringes are of maximum visibility. With most practical sources of monochromatic light the visibility of the fringes falls off fairly rapidly when the path difference is great, so that the adjustment of the mirrors is usually made to secure the most distinct fringes. For this case $x=t(\mu \cos r-\cos i)=t\times 0{\cdot}638$, and equation 2($a$) becomes

$$R=\delta^2 t(0{\cdot}334+0{\cdot}638+0{\cdot}266 \cos {}^2\varphi).$$

The ellipticity of the fringes is $\delta_v/\delta_h=\sqrt{1{\cdot}238/0{\cdot}972}=1{\cdot}13$. The horizontal radius of the first fringe (when centre is bright) is $\sqrt{\dfrac{\lambda/2}{1{\cdot}238\,t}}$ radians or $0{\cdot}0046 f/\sqrt{t}$ measured in the focal plane of a lens. For the values of f and t assumed earlier, this is just about 1 mm.

Effect of a Second Glass Slab.—Reverting to the case in which the mirrors are equally distant from the reflecting surface, if we insert another slab in one of the beams, the resulting retardation is the sum or difference of that due to each. If the second slab is of thickness t', and the angle of incidence on it is i', and if it is placed in the beam which does not traverse the glass of the semi-reflecting plate, the resulting retardation for an inclination δ to the central ray is

$$\begin{aligned} R &= t\delta^2(A_{45}+B_{45} \cos {}^2\varphi)-t'\delta^2(A_{i'}+B_{i'} \cos {}^2\varphi) \\ &= \delta^2 \{A_{45}t-A_{i'}t'+(B_{45}\, t-B_{i'}t') \cos {}^2\varphi\} = \delta^2(a+b \cos {}^2\varphi). \end{aligned}$$

where $A_{i'}$ and $B_{i'}$ are calculated from the refractive index of the second slab.

The fringes may be either elliptic or hyperbolic, including as special cases straight lines or circles, depending on the signs and relative values of a and b.

For instance, in the case of a second slab of thickness 1·25 cm. and refractive index nearly equal to that of the interferometer plate (thickness 1·8 cm.) the fringes, when the plate is normal to the rays, are ellipses of which the vertical axis is approximately twice the horizontal. When the plate is turned from the normal position the ellipticity diminishes, and when i' is in the neighbourhood of 52° the fringes are circles. On further increasing the obliquity they again become elliptic, but with the long axis horizontal. At about 65° the ellipses have flattened out into horizontal straight lines, and at still higher angles they become hyperbolic.

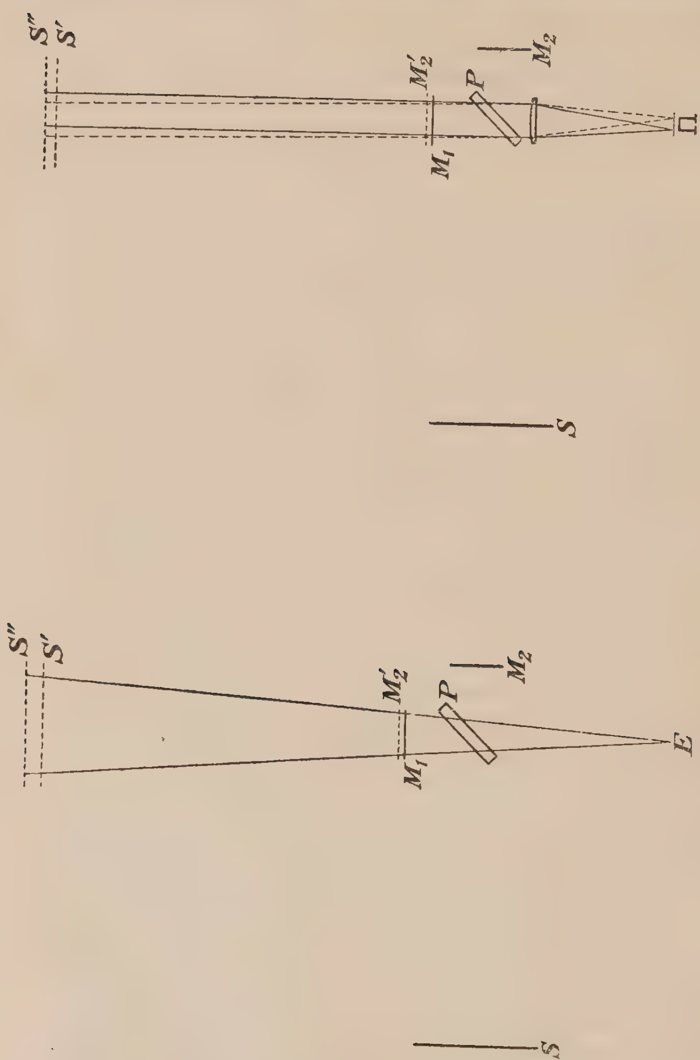
It is only if the plates are of the same material and thickness, and are equally inclined to the incident light, that a and b can be zero simultaneously, which is the condition for exact compensation.

The retardation equation when, in addition to a second glass slab in one of the paths, there is also a difference in the distances of the mirrors from the semi-reflecting surface, is clearly

$$R=\delta^2\{A_{45}t-A_{i'}t'\pm x+(B_{45}t-B_{i'}t')\cos^2\varphi\}\,.$$

Effects of Film Angle.—We have so far assumed the surfaces of the air film (to which the interferometer is equivalent) to be parallel. The effect of inclination of these surfaces depends on the method by which the fringes are observed. If they are simply viewed by an eye at E, Fig. 2 (a), so that the light reaching the eye from any direction passes through a particular point of the film, the effect of an inclination of the surfaces is to displace the centre of the fringe system, provided there is an air gap between M_1 and the image of M_2. The effect is due to the prismatic air film. It is shown in the text-books (for instance Mann, "Manual of Advanced Optics," page 53), that the centres of symmetry of the circular fringes observable with a thick film, when received by the naked eye (or a telescope of small aperture focussed on infinity), are only concentric with the normal from the eye to the film if the film is parallel. If the surfaces are inclined the retardation in a direction δ from the normal depends not simply on δ^2, but on δ^2+a term in δ^3. As a result the centres of the fringes are displaced from the normal. Similarly the addition of a term in δ^3 in equations 2 or 2 (a) produces a displacement of the fringes parallel to the direction of greatest slope of the film. If, however, the fringes are observed by a telescope of large aperture, so that for each point in the focal plane light *from all parts of the film* is utilised, as in Fig. 2 (b), this effect is absent; since, in these circumstances, the retardation at any point in the fringe system depends only on the inclination of the rays, and on the *mean* thickness of the whole film. The fringes are in this case simply a function of the inclination of the rays on the film, and give no information about the paralell-ism or flatness of its surfaces; except that unless they are nearly flat and nearly parallel the *distinctness* of the fringes will be affected on account of the confusion of phase at each point in the system.

Effects with Prisms and Lenses.—If we insert a refracting prism in one of the paths of the interferometer, and adjust the mirror so as to be perpendicular to the mean direction of the



(a) (b)

FIG. 2.

S Broad Source.
P Semi-silvered surface.
M_1, M_2 Interferometer Mirrors.
M'_2 Image of M_2 in P, forming with M_1 the equivalent film.
S', S'' Images of S in M_1 and M'_2.

refracted beam, the latter will be returned back through the prism, and, after reflection by the semi-silvered surface, will produce interference effects with the beam from the other

mirror. The excess of retardation of one of the beams is made up of a certain amount of extra glass path and a certain amount of extra air path, the former progressively decreasing and the latter increasing from the base to the apex of the prism. The resulting fringe system due to rays inclined to the mean ray is complicated by this fact, and also by the fact that the deviation varies with the angle of incidence and with the inclination of the ray to the principal plane of the prism. These latter peculiarities give rise to the result that an incident ray, inclined at an angle δ to the ray which ultimately meets the mirror normally, is not inclined to it at an angle $-\delta$ after returning through the prism, and consequently is inclined to the corresponding ray in the other beam after recombination at the semi-silvered plate. This is in effect equivalent to the interference film having a different angle between its surfaces for rays incident in different directions. As a result of these various factors, a general expression for the resulting fringe system would be of a somewhat complicated form.

It is easy to deduce, however, what the phenomena observed in the focal plane of a telescope must be. For each point in the field of view, light is received from all parts of the prism; and, conversely, each element of the prism supplies light to every part of the field. The resultant fringe system observed through the eyepiece is, therefore, the resultant of the systems corresponding to each element of the prism, which may be approximately regarded as elements of slabs of the appropriate thicknesses. The systems will vary in character from the base to the apex, in accordance with the variation in the relative glass and air paths. The only point in common is the centre of the systems ($\delta=0$) and its immediate neighbourhood. Thus, we should expect to see one or possibly two fringes at the centre of the field, the outer parts being uniformally illuminated owing to the confusion of the outer parts of the various component systems. This is, in fact, what may be observed in practice. Usually only the central fringe is visible; it is approximately circular, elliptical or hyperbolic, depending on which form predominates among the component systems.

When a lens is placed in one of the beams in conjunction with a coaxial convex mirror, situated with its centre of curvature at the focus of the lens, the beam after returning through the latter will produce interference with the other beam. The resulting effects are clearly similar in character to those deduced in the case of a prism.

Contour Fringes.—The fringes which we have discussed so far are due to the varying obliquity with which the rays corresponding to different parts of the field of view meet the various surfaces of the interferometer. In the case of naked eye vision there is, as we have seen, a slight displacement of the system if the equivalent film is prismatic, and there would be a slight distortion of their form if the surfaces were irregular; but if a telescope of large aperture is used, so that for any part of the field of view rays from the whole of the illuminated area of the film are utilised, the fringes are unaffected, except in distinctness, by either the inclination or flatness of the surfaces of the film. Such fringes are, of course, useless for testing the surfaces of the mirrors or the uniformity of optical path through a piece of optical apparatus placed in one of the beams.

For such purposes it is necessary to utilise the fringes due to a *point source* situated at infinity. Rays from such a source, which may be a very small pinhole at the focus of a telescope object glass, reach all parts of the equivalent film at the same inclination. If, when the beams have passed through the interferometer and have been recombined, they are collected by a second telescope objective, two images of the pinhole will be formed in its focal plane. If the film is nearly parallel these will coincide, and an eye placed at this point will see the whole of the film illuminated. Each point is seen by a separate ray; and, if the optical thickness of the film is uniform, it will appear uniformly bright or dark. If, however, the film thickness varies, the retardation for rays reaching the eye from different parts of it will vary, and fringes will be seen which are true contours of the film thickness; for, since δ is the same for all parts of the field, the retardation for any ray can only depend on $\Sigma \mu t$, the total optical path which the ray traverses, and is independent of what proportion of the path is in glass or air. Thus the contour fringes are not affected in form by the inequality of glass path in the two sides of the interferometer; but the conditions under which they will be distinctly visible are affected. There is obviously no such thing as a point source in practice: the pinhole must have an appreciable area in order that the field may be reasonably bright. We may, therefore, regard it as a small portion of a large source, and its image in the focal plane of the telescope objective as a small area of the fringe system which a broad source would produce there. If the light

meets the mirrors normally, the image will be in the centre of this system, where the retardation is varying slowly, and there will be no appreciable change in phase between one part of the pinhole and another. In these circumstances, the contour fringes seen when the eye is placed at the image will be most distinct. If, however, the light is not normal to the film, the image of the pinhole is located in an outer portion of the infinity fringe system, where the phase variation is rapid. The retardation, therefore, varies appreciably from one side of the pinhole to the other, and the eye receives light of a range of retardations from each point in the contour fringe system. As a result, the distinctness of the fringes is much reduced.

With an interferometer exactly compensated, and with the mirrors adjusted for equality of path, the fringes would be visible for all angles of incidence. Thus, one consequence of the lack of compensation is to render it necessary to adjust carefully for normal incidence in order to obtain clear contour fringes. The accuracy required varies as the diameter of the central fringe produced in the focal plane of the telescope with a broad source. When testing certain lenses, for instance, the normality has to be very exact or the contour fringes are scarcely visible. In other cases there is more latitude.

Another point to which attention has to be directed in using an uncompensated interferometer for contour testing is that the point source must be exactly at the focus of the collimating lens. If it is not so, the rays from the outer parts of the lens are inclined to the axial ray, and the retardation will vary from the centre of the field outwards, quite apart from contour variations. Excessive spherical aberration in the collimating lens will have the same effect.

ABSTRACT.

The Paper mainly consists of an investigation of the form of the fringe system observable at infinity, or in the focal plane of a telescope, when a broad source is employed with a Michelson interferometer, in which the glass paths of the two interfering beams are not equal. The fringes may be elliptical or hyperbolic, with circles and straight lines as special cases. In the recently developed method of using the instrument for optical testing, the fringes due to a point source at infinity are employed. It is shown that the form of the fringes in this case is unaffected by lack of compensation, but that the visibility of the fringes is conditioned by the nature of the fringe system due to a broad source.

DISCUSSION.

Mr. F. Twyman communicated the following remarks :—

As I shall not be present on the occasion of Mr. Guild's Papers on "Fringe Systems in Uncompensated Interferometers" and on "The Location of

Interference Fringes," I am writing to say how greatly I welcome the papers. The class of interferometers dealt with is becoming more and more used in the workshops with which I am connected, and it is of very great advantage to us to have the conditions for satisfactory distinctness of the fringes combined with good illumination clearly set forth.

So far as optical elements with flat surfaces are concerned, Mr. Guild's papers seem to me to provide everything that is needed in this respect.

I hope that at some future time Mr. Guild will find time to deal in a similar way with the fringes of the lens interferometers.* We are, of course, acquainted in a vague and general way with the desirable conditions for securing plenty of light simultaneously with good definition of the fringes, but the precise formulae are as yet not available.

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* F. Twyman, "Phil. Mag.," Jan., 1918, p. 49, and "Phot. Journ.," Nov., 1918, p. 239.

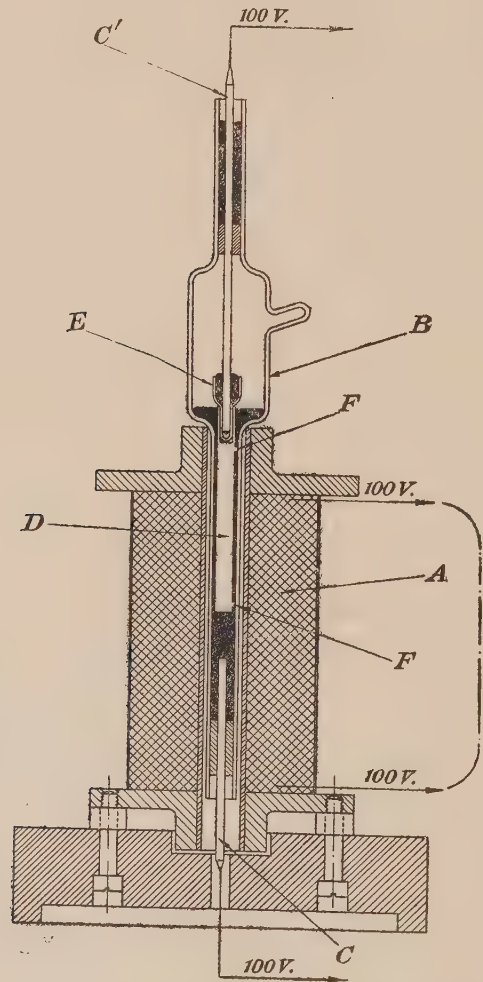
IV. *A New Relay for Moderately Heavy Currents.* By Guy Barr, *B.A., D.Sc.* (*From the National Physical Laboratory.*)

(Communicated by F. E. Smith, O.B.E., F.R.S.)

The need frequently arises in physical laboratories for a relay which will switch on and off currents of a higher order than can be dealt with by the simple Post Office or similar type in which an electro-magnet is made to bring two platinum terminals into contact. At the National Physical Laboratory a solenoid device has been in use for some years which makes a circuit by lifting an iron core attached to a lever which lowers a copper fork into mercury cups. This form has been in fairly constant use for cutting in and out currents up to 4 or 5 amperes on a 100-volt supply. If there is much inductance in the circuit, the arc produced on break, even with smaller currents than this, is, however, so pronounced that the mercury rapidly fouls and "splutters" out of the cups. The difficulties due to the use of higher currents could, of course, be largely overcome by the use of a step relay with a more powerful magnet, &c., but it has recently been found possible to construct a relay in which a solenoid operating with about 0·03 ampere at 100 volts is capable of breaking currents up to 20 amperes.

Its action depends on the fact that no arc can be maintained between mercury electrodes in hydrogen at atmospheric pressure, at any rate, with ordinary voltages. A first form of switch was made consisting of a short glass tube of about 1 cm. bore, provided with small-bore side-tubes near each closed end; the side-tubes were then bent into U-tubes and filled with mercury, the wider tube being also about one-third full of this liquid and the remainder full of hydrogen. Into the open arms of the U-tubes were led iron or nichrome wires from the circuit to be operated, and the tubes were then closed with sealing-wax. By tilting the apparatus the mercury in the wide tube could be made either to collect in a pool at one end or to form a continuous layer connecting the U-tubes. It was found that this gave very clean breaks with quite large currents (20 amperes), but trouble was experienced when it was attached to the lever of the usual solenoid relay. A much more satisfactory form was eventually designed, which will be most readily understood from a diagram.

The lower end of the glass tube *B* of the shape shown fits into the axis of the solenoid *A*; *B* is provided with iron leads *C*, *C′*, of about 0·15 cm. diameter, which are enlarged by collars fitting loosely into the ends of *B*. These iron



collars are cemented gas-tight in position with sealing-wax; the gas-tightness of the upper collar is further improved by a seal of mercury above the wax, and another layer of wax serves to contain the mercury. Moving in the part of *B*

which lies within the solenoid is an iron core D, carrying at its upper end a glass or silica cup E, a short tube projecting from which is inserted into a hole in the core. The lower end of the upper lead C' fits with a small clearance into the bore of the tube of E. Mercury fills B up to a suitable level, such that the rim of E is some 1 cm. clear of the mercury level when the core floats, and about 0·5 cm. below the mercury when the core is sucked down by the solenoid. The core does not fit the tube of B, but is provided at the bottom and at two-thirds of its length with three centering projections F; by this means a free passage for mercury is ensured and the friction against the sides is very small. A central position is further ensured when the core is floating by the fit of C' into the "tail" of E ; the fit here is intended to absorb some of the shock occurring when the glass cup is arrested in its quick upward movement by the end of the iron rod C'. The space above the mercury in B is filled with hydrogen introduced before the sealing of the small side tube shown. In use the cup E becomes filled with mercury and the break of the current occurs on the glass rim of the cup, *i.e.*, between mercury electrodes ; this is important since other metals are likely to produce fouling of the contact. The edge of E should be thin, to prevent cracking by the spark, or preferably it would be made of silica.

In the examples which have so far been made the core has been of mild steel ; this is quite satisfactory with a core 6 cm. by 0·4 cm., using a solenoid 8 cm. long containing $\frac{1}{2}$-lb. of copper wire of resistance 3,000 ohms. If electrolytic iron, stalloy or similar more permeable metal were used the dimensions of the core could be smaller or the solenoid weaker. The use of mild steel is further disadvantageous, since contact between the core and C must then be prevented, *e.g.*, by a small wooden plug, lest the remanent magnetism should cause sticking when the current through the solenoid is turned off. This precaution could probably be omitted if suitable soft iron were used.

One of these relays has been run for several days, switching a current of 17·5 amperes on and off at intervals of about a minute and the mercury has remained quite clean. The resistance of the system betweem C and C', with core sucked down, is about 0·01 ohms when a current of 10 amperes or more is running ; half of this is accounted for by the iron leads. When the instrument has been out of action overnight, the contact resistance between the iron wire and the

mercury rises, in one model which has been examined, to 0·2 ohms for a current of 1 ampere, but this resistance rapidly falls to the normal low value when the current is increased. With a larger model the resistance was initially about 1 ohm for a current of 0·1 ampere, and fell to 0·18 ohms at 1 ampere and to ·01 ohms at 10 amperes; on then passing the smallest current the resistance remained 0·01 ohms.

The rise in resistance is presumably due to the formation by diffusion of a thin film of gas between the iron and mercury; when a current flows the thickness of this film would be rapidly reduced, owing to the heat developed at the junction, and to the rotation of the mercury in the magnetic field, which would assist the escape of bubbles.

The author would like to take this opportunity of thanking Mr. G. H. Glaysher for the interest he has taken in the development of this relay and for certain improvements suggested in details of mechanical construction of the final form exhibited.

ABSTRACT.

The action of the relay depends on the fact that no arc can be maintained between mercury electrodes in hydrogen. One lead is brought to mercury contained in a vertical tube within a solenoid. An iron rod, at the upper end of which is a glass cup, floats in the mercury. The cup also contains mercury, and the other lead is connected to an iron rod which dips into this. When no current flows in the solenoid, the rim of the cup is about 1 cm. above the level of the main body of mercury. When the relay current (about 0·03 ampere) is running the iron rod is sucked down until the rim of the cup is submerged by about 0·5 cm. The space above the mercury contains hydrogen. The relay can be used to break quite large currents (20 amperes) without much spark.

DISCUSSION.

Mr. C. C. Paterson asked on what voltage the apparatus would work, as the energy in the spark would depend on this.

Dr. Barr said he had not tried it above 100 volts.

Mr. F. E. Smith suggested as a simplification that a U tube might be employed, in one limb of which was the iron float. When this was sucked down the rising mercury in the other limb of the U could make connection between two iron electrodes.

Dr. Barr said it was important that the break should take place between mercury surfaces, otherwise the mercury got contaminated.

Mr. Guild suggested that if the mercury in the second limb of the U were made to flow over and make connection with some stationary mercury in a side tube the difficulty of Mr. Smith's arrangement would be overcome.

Mr. Paterson asked how hot the apparatus would get if the current were broken as frequently as, say, 10 or 20 times per minute.

Dr. Barr replied that he had not had occasion to try it as rapidly as that.

Dr. Hopwood asked if the break was "clean," or if the oscillations set up in the mercury were sufficient to make and break the current after the first break had occurred.

Dr. Barr said there was no evidence of this. There was very little disturbance of the mercury in actual practice.

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